

Effective Practices in Post-16 Vocational Maths

Final Report

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THE RESEARCH  BASE

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The Research Base is an international social research consultancy specialising in education and skills.

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Acronyms

Acronym	Full Version
ACME	Advisory Committee on Mathematics Education
ACOLA	Australian Council of Learned Academies
AQA	Assessment and Qualifications Alliance
ATM	Association of Teachers of Mathematics
BIS	Department for Business, Innovation and Skills
BTEC	Business and Technician Education Council
CPD	Continuing professional development
DfE	Department for Education
EMCETT	East Midlands Centre for Excellence in Teacher Training
ESOL	English for speakers of other languages
FE	Further education
FSMQ	Free standing mathematics qualifications
IMA	Institute of Mathematics and its Applications
JMC	Joint Mathematical Council
LLN	Language, literacy and numeracy
LSA	Learning support assistant
MaSE	Mathematics and science education
MEI	Mathematics in Education and Industry
MLP	Mathematics linked pair
NANAMIC	National Association for Numeracy and Mathematics in Colleges
NCETM	National Centre for Excellence in the Teaching of Mathematics
OECD	Organisation for Economic Co-operation and Development
OFSTED	Office for Standards in Education, Children's Services and Skills
PET	Professional education and training
PISA	Programme for International Student Assessment
RAMR	Reality, abstraction, mathematics and reflection
RME	Realistic maths education
RSA	Royal Society for the encouragement of Arts, Manufactures and Commerce
STEM	Science, technology, engineering and mathematics
TIMSS	Trends in International Mathematics and Science Study
VET	Vocational education and training
VLE	Virtual learning environment

Foreword

Maths and English are the key enabling subjects. At the Education and Training Foundation, we have been thinking for some time about how we can support maths provision to be the best it can be, and what we can learn from others nationally and internationally. In that context, working with the Department for Education and the Cabinet Office, we commissioned this research to look at some of the best ideas from around the world for teaching maths to learners aged 16-19.

A strong message comes through that re-taking GCSE (or a similar qualification) is not the best route for everyone. There are more mixed messages on embedding maths in a vocational area: while 'all' or 'nothing' both seem to work as approaches, doing a bit of embedding and a bit of stand-alone maths can cause confusion. There are also complex issues around institutional transition, progression and the reliability of the GCSE grade as a signal of numeracy which must be addressed: diagnostic assessments at age 16+ are leading to some learners being put onto provision below the level which their GCSE Grade might suggest would be the correct, stretching next step.

The Foundation has now been asked by the Government to lead a review of non-GCSE English and maths provision in England, and how well it equips learners for success at work. This timely report will enable us to base this review on effective international practice.

David Russell
Chief Executive, Education & Training Foundation

December 2014

1. Executive Summary

1.1 Study Aims

The objective of the project is to inform work which aims to improve the teaching and delivery of maths for learners aged 16 to 19, with a focus on the teaching of learners who have previously struggled with maths and who have reached the age of 16 without a GCSE in maths at grade C or above. It looks particularly at innovative approaches and international good practice in this area, and focuses on pedagogy as well as how to enhance learner motivation and engagement. It also considers the experience of employers in supporting the teaching of maths to learners who are working and studying post-16.

1.2 Methodology

Research was conducted in four phases:

The UK literature review identified areas of effective teaching of maths, based on the particular circumstances and support needs of the target group. Three areas in particular were prioritised: innovative methods of teaching, measures that enhance learner motivation and the experience of employers in supporting the teaching of maths - especially those who take on apprentices aged 16 to 19.

The international review investigated effective practices in the teaching of post-16 vocational maths in countries which have performed well on international indices. It further reviewed wider policies and practices which may have contributed to positive learning outcomes in these countries, and identified any policies and/or practices which might be applicable in the UK context. Countries used as case studies were Shanghai-China, Singapore, South Korea, Japan, Switzerland, the Netherlands, Estonia, Germany, Canada, Australia, Ireland and New Zealand.

A call for evidence was circulated using a wide array of channels, including social media, ETF's newsletter, and contacts of the Research Base, MEI and the Policy Consortium. Submissions were received from a range of maths specialist organisations, further education (FE) providers and individual consultants.

The UK qualitative review considered current challenges and best practices in post-16 vocational maths education across a range of providers, employers and maths specialists. Qualitative interviews (a mixture of face to face and phone interviews) were held with 20 employers, 20 providers and 11 maths specialist organisations.

Limitations focused on the lack of availability of quantitative and large scale material for the UK review; similarly, there was a paucity of information available for the international review.

1.3 Key Findings

Maths Skills in the UK

Almost half of young people do not gain at least a C grade at GCSE maths, and three quarters of full-time students who have not achieved a maths GCSE at grade C or above by the age of 16 enter FE colleges. Differences between initial assessments in vocational settings and previous GCSE assessments means that some post-16 students in vocational education may be studying maths below the level they have already attained.

Employers' requirements vary from basic applied maths skills to conceptual abilities depending on the context. As a result, required maths skills and procedures are often embedded within highly individualised practices developed and adopted within each workplace. Despite additional sectoral differences, standard minimum requirements linked to

GCSE maths are common, but employers often find that young people are unable to apply mathematical concepts they have learned in school to novel problems in the workplace.

International Policy and Practice

Cultural context contributes to international differences in maths pedagogy, learners' progression and broader societal attitudes towards the learning of maths. The success of vocational maths policies is affected both by this cultural importance given to maths, and by the success of pre-16 maths teaching. Factors such as long study hours may, however, mask ineffective policies regarding maths learning in vocational settings in comparator countries.

At the upper secondary level, countries tend to prioritise either maths concepts, focusing particularly on in-depth knowledge of curriculum content, which tends to be part of a policy agenda to gain momentum in the PISA/TIMSS rankings; or generic competencies, linked to a policy focus on maths skills for economic development.

Equity has a positive association with overall achievement and attainment in maths. Shanghai has had particular success in ensuring depth of success in mathematics across the whole student population, not simply among the most talented. The small gap between the highest and lowest performers and the importance given to equality as well as quality has been identified as a key factor behind the city's success by the Shanghai PISA centre.

The review of international practices demonstrates that no one single approach is appropriate for learners; approaches must be combined and tailored according to the specific needs of the learners being taught. There are, however, approaches that could be adapted to, and useful for, the UK context. Certain pedagogical models have been highly successful in the post-16 vocational context. These include the Canadian approach of Discovery Math that encourages learners to apply problem solving skills to mathematical problems; the Australian Reality, Abstraction, Mathematics and Reflection (RAMR) Model, which centres on problem solving and creativity, linking students' current knowledge to their social and cultural backgrounds, and the use of symbols; and Realistic Mathematics Education (RME) in the Netherlands which emphasises the use of real-world examples, contextualisation and activity-based learning.

The Application of Maths

It is important, where curricula allow, to tailor content to specific learners' needs. Disengaged learners, particularly those who have had negative learning experiences in the past, may need to be supported to develop an identity through which they can view themselves as being competent to deal with real-life situations, such as money, employment and the law. Within the constraints of nationally set curricula, however, teachers and trainers have more freedom over their teaching methods and approaches than they do over content. Countries where teachers have greater autonomy over how to interpret the curriculum, and therefore tailor it to the individual learner, tend to experience good outcomes in maths.

Engaging Learners

Many young adults on vocational courses can be disengaged from and have negative attitudes towards learning maths; this can be affected by multiple factors, including negative prior experiences with learning, peer pressure and lack of confidence. Family income level is additionally associated with the extent to which learners have negative attitudes towards maths, with poverty showing a strong correlation to mathematical anxiety. Maladaptive beliefs about maths learning and low self-esteem can make learner engagement particularly challenging for teaching staff. Changing mindsets so that learners develop confidence and 'mathematical resilience' is often key to re-engaging young people with maths.

The reasons that learners disengage from learning are diverse, and therefore strategies to reengage them with learning must be equally so. There is no one easy solution and teachers must be supported - through training and CPD - to have a range of tools available in order to ensure a range of available approaches. Diversity in classroom approaches can be underpinned by provision of a safe and organised environment, sufficient resources and a sense of community.

Embedding Maths into Vocational Contexts

A variety of literature supports the idea that contextualised learning of maths improves both learners' overall understanding and the extent to which they retain the information they have learned. Piecemeal contextualisation, however, can be limiting for learners; ensuring that contexts are planned and aligned to the broader curriculum, and that contextualised mathematical tools are applied across several areas, can help to avoid this.

Maths in the Workplace

Apprenticeship and other workplace-based learning programmes enable young people to apply the skills they are learning to real-life contexts; confidence and engagement levels are supported by the relevance of the workplace itself. Flexibility and one-to-one support and mentoring programmes can help learners who are struggling; for example, flexibility around delaying the point at which apprentices start studying maths. Other areas which can support previously disengaged learners to succeed in workplace-based maths include giving them ownership of the learning process and ensuring that a good rapport is built with teachers and trainers.

Professional Skills and Training

A lack of skilled teachers and trainers in vocational maths settings has been a key challenge, although work done by the Maths Enhancement Programme is expected to improve the pipeline of qualified teachers. Motivation levels can be supported by collaborative working; this includes peer lesson observation, the sharing of ideas and collaborative

planning. Collaborative working between teachers tends to be most successful when supported by a whole organisation approach which incorporates a strong numeracy strategy.

1.4 Policy Recommendations

Progression

Building learner confidence and challenging maladaptive beliefs so that learners can engage with maths is vital. Ways to build confidence include mentoring, one to one support, clear progression routes and ensuring that provision is directly tailored to the learner's current level.

Although Functional Skills are viewed as stepping stone qualifications to GCSE, they were not designed for this. Currently, the level 2 Functional Skills qualification does not enable progression to GCSE maths in a structured way, nor does it lead to more applied higher level maths such as the level 3 Core Maths qualification. It is a smaller and shorter qualification, designed to be flexible and fit around the requirements of a vocational study programme.

The development of a qualification that could be completed in a modularised way as a series of small units, but which leads to a higher level qualification such as level 3 Core Maths, could provide a clearer progression route. A level 2 Core Maths qualification that draws on the applied nature and modularised route of the Functional Skills qualification, as well as the content of GCSE maths to enable progression directly onto level 3 Core Maths, may be a solution to these issues.

Curriculum and Assessment

Diagnostic assessments, whether before or at the beginning of a course, are important to ensure that learners are placed on appropriate programmes and are progressing well. Many initial assessments suggest results which are significantly different from learners' existing qualifications; learners are often found to be learning at a different level than their GCSE attainment suggests. Assessments must be designed in a way which supports learners and builds their confidence levels; further research around appropriate assessment mechanisms and current innovations in the area would be worthwhile.

A maths expert group could assure the quality and robustness of such an approach. Such a body could also make recommendations for maths in the national curriculum. Changing learner mindsets towards maths is vital, and the newly established body could work towards Core Maths content that is highly engaging and that motivates learners.

Support for Teachers

Additional pre-service training and CPD for teachers and trainers, both in maths and in how to contextualise maths, is likely to improve outcomes for learners. Research reviewed for this study found that specialist support for contextualisation - for example, having a maths specialist available to support contextualisation in construction training - improves outcomes. CPD requirements therefore include support for maths specialists in methods for contextualisation.

2. Methodology

The study aims to inform policymakers and practitioners engaged in the teaching and delivery of post-16 vocational maths. There is a particular focus on how best to support learners who have previously struggled with maths and who have reached the age of 16 without a maths GCSE grade C or above. Key areas explored include innovative approaches to maths teaching and learning; international good practice; measures that enhance learner motivation and engagement; and recent pedagogical developments. The study also considers the experience of employers in supporting maths teaching and learning for those working and studying post-16.

Research was conducted in four phases:

The UK literature review identified areas of effective teaching of maths, based on the particular circumstances and support needs of the target group. Three areas in particular were prioritised: innovative methods of teaching, measures that enhance learner motivation and the experience of employers in supporting the teaching of maths - especially those who take on apprentices aged 16 to 19.

The international review investigated practices in post-16 vocational maths in countries that have performed well on international indices. It further reviewed wider policies and practices which may have contributed to positive outcomes in these countries, and to identify any policies or practices which might be applicable in a UK context. Countries were selected on the basis of their PISA maths scores. The review looked at the top two cities/city states in the rankings (Shanghai-China and Singapore), the top two Asian nations (Korea and Japan), the top four European nations (Switzerland, the Netherlands and Estonia; with Germany substituted for Finland at ETF's request) and the top four Anglosphere countries (Canada, Australia, Ireland and New Zealand). Where possible, papers were reviewed in original languages as well as English; this was the case for Korea, Switzerland, Germany and the Netherlands. Due to the overall paucity of information available in the international review, a wider review was also conducted beyond the original 12 countries. Individual country case studies are provided separately to this report.

The call for evidence was circulated using a wide array of channels, including social media, ETF's newsletter, and contacts of the Research Base, MEI and the Policy Consortium. Submissions were received from a range of maths specialist organisations, further education (FE) providers and individual consultants.

The UK qualitative review considered current challenges and best practice in post-16 vocational maths education across a range of providers, employers and maths specialists. Qualitative interviews took place with 20 employers, 20 providers and 11 maths specialist organisations, with sampling as follows:

Qualitative Interview Sampling	
Providers (x20)	20 providers were interviewed, covering all regions in England. Five respondents represented colleges rated as poor performing in maths, with the remainder being those identified as high performing in maths based on official OFSTED figures. Five of the 20 providers covered were also independent institutions, with the remainder being further education colleges.
Employers (x20)	20 employers were interviewed across the following sectors: construction; hair and beauty; health and social care; catering and hospitality; land-based services; technology; transport; manufacturing; retail; business and administration; sports and leisure; the Army; and the Navy. Five employers interviewed represented small to medium sized companies, with the remainder representing medium to large businesses. Three were public sector, with the remainder in the private sector.
Maths specialist organisations (x11)	<ul style="list-style-type: none"> • The Advisory Committee on Mathematics Education (ACME) • The Association of Teachers of Mathematics (ATM) • The Centre for Innovation in Mathematics Teaching • The Department of Education, University of Oxford • The Institute of Education's Department of Lifelong and Comparative Education • The Institute of Mathematics and its Applications (IMA) • The Joint Mathematical Council (JMC) • The Mathematics and Science Education (MaSE) Research Centre, University of Southampton • The National Association for Numeracy and Mathematics in Colleges (NANAMIC) • The National Centre for Excellence in the Teaching of Maths (NCETM) • National Numeracy

The literature reviews were conducted online using a pre-defined search strategy and search terms. Interviews were conducted both on the telephone and face to face, using a semi-structured interview guide tailored to each respondent group. Responses were recorded, transcribed and anonymised for later analysis. Individual research phases were developed into draft reports, which were later triangulated to inform the final analysis.

2.1 Limitations

The majority of material available for the UK review was qualitative and small-scale. Areas of positive impact may therefore be related to the abilities and approach of individual teachers, the motivation and engagement levels of the learners whom they are teaching, and the relationships developed, rather than the stated study outcomes themselves. Without larger scale studies in this area, generalisations are difficult; examples of practice should therefore be read and interpreted with a view to the small scale studies on which they are based. It should also be noted that the policy landscape is evolving rapidly; there have been changes since project inception and there may be further ones prior to publication.

The paucity of available information for the international review was another major limitation. This, in our view, is due to a number of factors, including: the success of pre-vocational school-based maths in most of the study countries, which removes the need for additional or remedial action at a vocational level; the cultural context within many of the study countries, in which mathematical success is highly valued, additional out-of-school tuition is common and disengagement of learners is unusual; and the significant priority given to school-based research over and above research linked to vocational institutions in all countries of study. It should be noted that the international review was intended to provide only a snapshot of relevant policies and pedagogies. The time available for research for each country was very limited; we recognise that there may be key texts and policy/practice examples which are missing.

It should finally be noted that in no circumstances can policies or practices be lifted from one context and transferred wholly into another, as there are complex relationships with culture, history and norms of the country in which they originate. We have borne this fact in mind whilst performing the summaries and analyses. Cultural and policy context is explored in more detail in chapter three.

2.2 Key Concepts

There are a number of key concepts for the discussion of post-16 vocational maths teaching and learning. While there is some degree of overlap between certain concepts, all these concepts are commonly employed by maths specialists and can be found in relevant literature.

Mathematics versus numeracy: The distinction between maths and numeracy is subject to some debate within current literature, in particular, the extent to which numeracy may be defined as applied mathematics. For the purpose of this report, we have used the definitions developed by National Numeracy, with numeracy defined as 'the ability to

use *maths in real life*’ and traditional classroom maths as ‘*mathematics beyond numeracy, for example, calculus, quadratic equations, statistical analysis*’. It should be noted, however, that where reports mentioning numeracy are quoted, the authors may have made a different distinction.¹

Functional Mathematics: Functional Mathematics qualifications are designed to equip learners with the basic maths skills required for their everyday personal and professional lives. It is available at levels 1, 2 and 3 and may be used as a stepping stone to the GCSE qualification.

Mathematical literacy: Mathematical literacy has been defined by PISA as ‘*an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen*’.²

Mathematical anxiety: Mathematical anxiety (MA) has been defined, according to the Nuffield Foundation, as ‘*a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in ordinary life and academic situations*’.³ It can severely affect the performance of both children and adults, and can lead to a situation in which those with MA actively avoid any situation which might require the application of maths.⁴

3. The Context

The following section summarises research findings regarding the overall position of maths skills in the UK, looking both to the learning of maths and maths skills in employment. The learning picture notes the low levels of functional numeracy amongst young people in the UK, the high proportion of students who have previously struggled with maths learning in the vocational education sector and evidence of poor targeting of learning programmes and qualifications. The employment picture discusses the needs for maths skills in the workplace, sectoral differences in maths skills requirements and notes the inability of many workers to apply mathematical concepts to novel problems in their place of work. Section three further summarises the cultural context of maths learning, noting the negative culture of maths learning in the UK.

3.1 Maths Skills in the UK

The Learning Picture

▶ **Many young people have low levels of functional numeracy.**

The maths skills of the English workforce are increasingly lagging behind competitor nations. Recent research suggests that one fifth of 16 to 19 year olds are ‘*functionally innumerate*’, with little more than basic arithmetic skills, and that this figure has remained largely static over the last 20 years. In this respect, the UK compares poorly to many other industrialised countries.⁵

▶ **The majority of young people who do not obtain a good maths GCSE go into FE colleges.**

Slightly under half of young people do not gain at least a C grade at GCSE maths,⁶ while around three quarters of full-time students who have not achieved a maths GCSE at grade C or above by the age of 16 enter FE colleges.⁷ This suggests the critical importance of good maths education in vocational settings.

▶ **There is evidence of poor targeting of learning programmes and qualifications.**

Participation and provision in vocational education is poorly documented and thus poorly understood. In England, some post-16 students in vocational education may be studying maths below the level they have already attained.⁸ The National STEM Centre has blamed this current state of affairs on provision being determined by ‘*non-demand incentives such as funding, targets and league tables*’.⁹ In addition, the number of students taking maths qualifications fluctuates significantly from year to year, suggesting that provision may be driven more by institutional factors (e.g., funding, targets, or league tables) rather than student demand or need, which may further exacerbate the problem of understanding participation and motivation.¹⁰

The Employment Picture

▶ **Employers’ requirements vary from basic applied maths skills to conceptual abilities.**

The level of maths required to perform different occupational roles has increased significantly over the last 20 years. The need for good, basic maths skills in the workplace, as well as a more conceptual understanding of maths, is widely recognised. A 2011 report by the Advisory Committee on Mathematics Education (ACME) noted that ‘*in the workforce there is a steady shift away from manual and low-skill jobs towards those requiring higher levels of management expertise and problem solving skills, many of which are mathematical in nature*’.¹¹ Other analyses also point to higher levels of maths skills required in lower and intermediate level jobs. Consequently, school leavers must possess sufficient mathematical knowledge and be able to apply that knowledge within the workplace.¹² CBI identifies several core skills required for ‘*functional numeracy*’ as demanded by employers in the workplace:¹³

- Mental arithmetic without the use of a calculator;
- The ability to interpret and respond to quantitative data;
- The ability to calculate percentages and interpret their significance;
- The ability to work comfortably with fractions, decimals and ratios;

- Awareness of different measures and the ability to convert between them;
- Instinct to pause and check potentially rogue results and calculation errors;
- Basic understanding of odds and probabilities.

► **Employees' ability to apply maths is often found by employers to be insufficient.**

While there is a demand for increased rigour in the teaching of the mathematical concepts underlying functional numeracy,¹⁴ a key concern amongst employers is that workers are able to apply mathematical concepts to novel problems in the workplace.¹⁵ The ACME report noted high levels of concern that the delivery of school maths suffered from an excessive emphasis on passing examinations, rather than teaching the application of mathematics to a variety of real-world problems.¹⁶ Hogden and Marks also noted that *'the 'skills deficit' is less about mathematical and calculation skills narrowly and more about how mathematics is applied and interpreted'*.¹⁷

There is, moreover, variation regarding numeracy requirements in different places of work. The numerical skills needed by employers are typically context specific, as a result of which required maths skills and procedures are often embedded within highly individualised practices developed and adopted within each, individual workplace.¹⁸ There are, furthermore, sectoral differences; though familiarity with mathematical software in general is becoming increasingly important to employers, there are relative differences between sectors.¹⁹ A 2011 mathematics task force report noted that employers indicated insufficient levels of mathematical competence amongst new employees; as a result, many organisations provide *'catch up'* courses in relevant numeracy skills.²⁰

► **Many work opportunities depend on having a grade C or above at maths GCSE.**

Although that level of maths may not be required on the job itself, employers report that many opportunities for work and progression depend on having a grade C or above.²¹ Maths remains, therefore, a critical skill for all, including those who have not achieved a minimum C grade at GCSE level by the age of 16.

3.2 The Cultural Context

Cultural context is critical as it affects the extent to which pedagogy and practices can be transferred to other situations. In the Nuffield Foundation's study of maths education in high-performing countries, it states: *'Implicit goals arising from nations' different socio-cultural-historical backgrounds have a powerful influence on pupil attainment. These implicit goals make 'borrowing' policies and practices problematic because adopting practices from elsewhere might mean adopting implicit goals that do not fit with England's vision for society or individuals. For example, the Asian emphasis on effort, while a factor in high attainment, is also closely aligned with obedience and authority.'*²²

► **Cultural context affects both the teaching of maths and learners' maths development; and cultural differences are visible in attitudes and values attached to maths.**

Maths is now acknowledged to be affected by the culture in which it is taught.²³ Culture is important both to the way in which maths is taught and to the way in which learners develop mathematical understanding and concepts. In Japan, for example, self-motivation of learners is encouraged by giving them greater control of tasks and through teachers valuing effort to a greater extent than ability, whereas in the United States it is the role of teachers to make lessons interesting and engaging.²⁴

In many countries, private tuition is becoming increasingly common. This ranges from private support for learners who have struggled with maths skills, as in Estonia,²⁵ through to the ubiquitous cram schools seen in Korea and Japan. In Japan, almost two thirds of eighth grade students receive private tutoring in mathematics outside school time. It is claimed that this tutoring complements the more conceptual teaching in Japanese schools by providing repetition.²⁶ In Japan, parental expectations and a culture which values success in maths also contribute to a high level of mathematical understanding among young people.²⁷

► **Family income level is associated with the extent to which learners have negative attitudes towards maths.**

Family income level is a key predictor of the extent to which learners will feel negative towards maths, with poverty showing a strong correlation with *'mathematical anxiety'*. Parents' education levels and their own attitudes to maths also influence the extent to which their children will feel negative about maths.²⁸

► **There is a negative culture of maths learning in the UK.**

Maths specialists interviewed noted that there exists a negative culture around the learning of maths, whereby the general perception is that maths is studied to GCSE level at the age of 16, and then is not studied again unless the learner is preparing for GCSE retakes or is undertaking a maths-based course. It was further noted that acceptance of failure in maths is a cultural issue. One maths specialist, for example, commented that *'in our culture it's okay to say, "I'm no good at maths" - it's not going to be easy to change the culture'*. There were also said to be cultural restraints that have been placed on young people: *'we do young people a massive disservice collectively as a culture by suggesting that you can become successful without any effort - that's just utter nonsense'*.²⁹

Singaporean students are, on average, two years ahead of their counterparts in the United States by the end of the sixth grade; this presents significant advantages once those learners move into the vocational stream.

- Centre on International Education Benchmarking

4. Policy Development

Section four examines the UK policy context regarding maths learning, challenges stemming from the policy context and looks to international policy lessons. The UK policy context examines reviews, including the Wolf Report, that have informed current policy on post-16 vocational mathematics; key challenges stemming from the current system are also considered. Section 4.3 discusses several lessons learned from a review of the international policy environment.

4.1 UK Policy

A range of reviews have influenced UK policy on post-16 vocational maths in recent years; the most significant of which is the Wolf report. Alison Wolf, who was commissioned in 2011 to review vocational education in England, found the country to be *'effectively unique in not requiring mathematics... for all young people engaged in pre-tertiary education'*.³⁰ Functional skills, which includes the embedding of maths teaching in vocational contexts, were subject to particular criticism: vocational teachers are not maths experts, and embedding maths in vocational contexts can lead to it disappearing from the teaching in any real sense. The report further concluded that functional skills were *'conceptually incoherent'* since they require multiple contexts and pedagogies in order to make them relevant to individual students. According to Wolf, this removes the possibility that they can be tested effectively through standardised assessment, a necessity of functional skills are to have any currency: *'this is not a circle which can be squared'*.³¹

The Wolf report recommended that GCSE Maths should remain compulsory until students achieve a grade C. The report advocates, however, that learners should be given an extra year to prepare for a retake. This extra year would allow students to develop a broader mathematical fluency and to avoid the sense of maths as a remedial activity.³² Wolf also recommended removing the barriers to free-standing maths qualifications, including funding policies and incentives that encouraged schools and colleges to focus on key skills and functional skills. She further suggested that individuals who have not gained a grade C and above in GCSE maths should not necessarily be required to take the qualification again immediately, recommending that other qualifications could be used as *'a pathway'* to GCSE - and, in some cases, in place of it.³³

The increased emphasis on strategies for effective maths delivery was reflected in Ofsted's revised Common Inspection Framework, implemented from September 2012 and updated in April 2014.³⁴ This included prompts that enable inspectors to evaluate how well maths teaching supports students in achieving their learning goals and career aims; the extent to which students understand the importance of maths; and whether staff possess the necessary qualifications to teach maths effectively. The first annual report produced under the new framework highlighted the tension between the high priority placed on maths skills by employers and the challenges faced by learning providers working closely with learners who failed to achieve level 2 maths at school.³⁵ Nonetheless, Ofsted has emphasised the key role that the FE and skills sector can play to help improve the English and maths skills of post-16 learners.³⁶

These reviews have informed current policy on post-16 vocational mathematics, key elements of which include the following:

Study programmes: The Government's response³⁷ to the Wolf Review gave impetus to the creation of Study Programmes.³⁸ These are designed to *'give all young people the opportunity to achieve the qualifications, skills and work-related activity, including work experience, needed for successful progression into employment or higher education'*.³⁹ Learners aged 16 to 19, taking both academic and vocational routes, are required to follow a Study Programme tailored to their existing educational level and intended career. As well as major qualifications and work experience, a vital expectation of Study Programmes is that learners who have not previously achieved at least a grade C in Maths GCSE should continue to work towards that level.⁴⁰

GCSE maths: A GCSE Mathematics Linked Pair (MLP) was piloted in 2011. This was a double award GCSE incorporating applications of maths and methods in maths. Both GCSEs had an emphasis on *'problem-solving, functionality and mathematical thinking'*.⁴¹ An evaluation of the MLP suggested that the teaching approaches required to deliver the paired qualifications were effective in enhancing students' experience of learning maths and increasing mathematical understanding.⁴² The level of *'engagement and motivation'* among students was reported to have increased.⁴³

The Department for Education published its response to the consultation on reforming GCSE subject content⁴⁴ in the autumn of 2013.⁴⁵ The Government confirmed that it preferred to work towards introducing a single, bigger GCSE maths qualification over the MLP.⁴⁶ The revised subject content for GCSE maths is intended to develop a knowledge of maths that would support problem solving in both real world and mathematical contexts, including the basic mathematical skills required in the workplace and for everyday life.⁴⁷ The implications for changes in pedagogy highlighted through the MLP developments would nevertheless be worthy of further exploration. This is reinforced by the Nuffield Foundation's emphasis on the need for *'mathematical fluency'* in supporting the progression of young people,⁴⁸ fluency in this context referring to the ability to apply mathematical skills to vocational and real life settings.

Free-standing mathematics qualifications: A series of Free-Standing Mathematics Qualifications (FSMQs) were previously piloted at levels 1 to 3; these qualifications were intended to *'bridge the gap'* between GCSE and A levels.⁴⁹ The focus on applied maths also helped students who had not passed GCSE maths to prepare for further vocational qualifications.⁵⁰ In 2010, Ofqual removed the GCSE-level designation from these qualifications and instead denoted

them as a 'Certificate in the Use of Mathematics';⁵¹ critics felt that the removal of the GCSE brand, however, could damage the credibility of the qualification and have a negative impact on the progression of post-16 learners.⁵²

Apprenticeships and traineeships: Apprenticeship providers are now required to offer a level 2 in functional maths or GCSE maths to learners without a previous level 2 qualification in maths. There is also a growing expectation that learners who have already achieved level 2 in functional maths will be able to study towards GCSE maths.⁵³ Level 1 remains a core requirement to achieve an intermediate apprenticeship.⁵⁴

The DfE and BIS updated the 'Framework for Delivery' for Traineeships in July 2013.⁵⁵ Maths is part of the six week to six month programme for Traineeships,⁵⁶ designed to support progression into Study Programmes.⁵⁷ The DfE and BIS also hope that apprentices of all ages will be working towards achieving the reformed GCSEs by 2020, and intend to work with employers to enable them to provide this.⁵⁸

Maths as a new requirement for post-16 funding: The teaching of maths qualifications became a condition of FE funding from September 2013, to be implemented from September 2014.⁵⁹ Functional skills and Free-Standing Maths will constitute a level 2 'interim qualification' as a stepping stone towards achieving GCSE maths at grade A* to C as part of the programme of study.⁶⁰ Several experts and studies have challenged the suitability of studying towards GCSE maths as a requirement for post-16 learners who have previously failed to achieve at least a C grade. In addition to its potential to demotivate learners (see section 4.2 below), the usefulness of the qualification for employment has been questioned. A study by Hodgen and Marks in 2013, for example, found that employees now tend to require an ability to apply basic maths in complex situations,⁶¹ meaning that swathes of the current GCSE curriculum are not relevant to employment practices.

4.2 Challenges in UK Policy and Qualifications

► Initiative and policy fatigue is prevalent among practitioners.

Policy churn is the biggest challenge facing further policy and practice supporting disengaged post-16 learners to study maths in the UK. Practitioners have dealt with wave after wave of new policies, updates to qualifications and teaching requirements. This makes it very challenging to implement further recommendations into a sector which is suffering from initiative fatigue.

Policy churn is the biggest challenge facing further development of policy and practice intended to support disengaged post-16 learners studying maths in the UK. Practitioners have dealt with wave after wave of new policies, updates to qualifications and teaching requirements. This makes it very challenging to implement further recommendations into a sector that is suffering from initiative fatigue.

One interviewee, for example, noted, *'I've been teaching twenty odd years now and it's literally almost every two or three years we've had a different qualification and what would be really useful is a bit of consistency, so that there weren't any more changes'*. Another commented on the need to be aware of and respond to a constant series of changes, even for those who are already doing a good job of delivering quality maths teaching. There was also some confusion over core maths, with one provider noting that *'the next issue that is going to come is the level 3, the core maths or whatever they are going to call it because that is the next big change. So we have got another year in preparation to be offering something like that. I'm not quite sure where that is going and who is going to be delivering that at the moment.'*⁶²

The RSA, in its 2012 review of maths at upper secondary level, found that *'too much diversity in qualification-type can be confusing for students, education-providers and employers'*; instead, it recommended that a greater breadth within the curriculum would offer better provision for different learning styles and needs.⁶³

► Political and financial stability are necessary to underpin the work of specialist maths organisations.

It was suggested that specialist maths organisations need regular and sustained funding, in addition to political stability, in order that they are able to continue their work supporting maths in the post-16 sector and supporting the policy objective of ensuring that maths is continued to the age of 18 for all learners by 2020.⁶⁴

► GCSE maths is potentially demotivating for post-16 learners.

Many experts and studies have challenged the suitability of studying towards GCSE maths as a requirement for post-16 learners who have previously failed to achieve at least a C grade. It has a high potential to demotivate learners who perceive themselves as having already failed. Evidence also suggests that it is counterproductive to push those FE students with a GCSE grade E or below to retake the GCSE course immediately.⁶⁵ Several experts commented that learner self-esteem and confidence could be positively affected by re-naming GCSEs or allowing a wider range of qualifications to be accepted as GCSEs.⁶⁶ Functional skills and FSMQs were said to be more engaging and motivating for learners, according to one maths specialist interviewed.⁶⁷ However, the difficulty in changing the mindsets of employers and universities, who require maths to a GCSE C grade, was also noted.⁶⁸

► There is a disconnect between GCSE pass rates and adult numeracy skills.

One maths specialist also commented that *'GCSE is not a good assessment of functional numeracy'*, as has been demonstrated through BIS and Skills for Life surveys. The BIS survey noted that GCSE pass rates stand at approximately 60%. The Skills for Life survey, which gathers statistics on competence in *'everyday maths'*, however, has shown that just 24% of 16 to 24 year olds are working at level 2. The interviewee noted, therefore, that there is a big disconnect between GCSE pass rates and adult numeracy skills.⁶⁹

▶ **The relevance of GCSE qualifications to employers is uncertain, and there is insufficient employer input into the GCSE maths curriculum.**

Maths specialists and providers interviewed called into question the relevance of GCSE maths for young people in vocational settings. There is an apparent disconnect noted between maths learned in school and maths required in daily working and personal life, and a challenge in ensuring that maths and numeracy teaching is relevant for employers. Employees now tend to require an ability to apply basic maths in complex situations,⁷⁰ meaning that swathes of the current GCSE curriculum are not relevant to employment practices. The lack of employer input to the development of curriculum content acts as a barrier to the usefulness of GCSE qualifications in vocational settings.

Interviewees further noted that employers look for GCSE qualifications but may not fully understand the relevance of other, alternative qualifications for work; it was suggested that *'what employers really want is a numerate workforce who are confident and competent with numbers - that's what we should be doing a better job of delivering'*. The same maths specialist actively promoted the development of a new maths qualification tailored to meeting employer needs.⁷¹

▶ **The GCSE maths curriculum is felt by specialists and providers to be outdated.**

According to our findings, the existing system does not prepare young people well for the requirements of maths in the modern world. One maths specialist commented on the *'disconnect between maths at school, which looks pretty similar to 150 years ago, and maths in the real world, which is driving Google, driving everything, driving iPads and Apples - it's all maths that is just in supply chains. The skeleton about which our world has made its maths and yet a huge swath of our population probably can't even look after their finances properly because they don't understand basic things about probability and everything else.'*⁷²

▶ **Attention given to numeracy skills has been insufficient.**

GCSE maths continues to be seen as the *'gold standard maths qualification'*.⁷³ Maths specialists expressed concern, however, that GCSEs provide an insufficient focus on numeracy; even those continuing with higher level maths may be functionally innumerate.⁷⁴ One maths specialist observed, for example, that level 3 engineering apprentices, where higher level maths is required, struggle with certain mathematical calculations: *'they didn't know what a third minus one was, and that's quite worrying - that they've come through school, the whole thing, and they've got their GCSE but the basics weren't there.'*⁷⁵

▶ **There is a lack of consensus over whether maths and numeracy should be separated at GCSE level.**

Interviewees differed in opinions regarding the need for separate maths and numeracy GCSEs. One maths specialist, for example, felt that they should be introduced separately after the success of the Welsh numeracy GCSE pilot. Another felt that separate qualifications were not needed and that the focus should be on a sound knowledge of maths. An alternative suggestion was that post-GCSE maths could be renamed and adapted to real-life situations.⁷⁶ Stepping Stones qualifications were also said to be useful for getting learners to work towards GCSEs.

▶ **Functional skills programmes of study may be more relevant for learners in vocational settings.**

Employers interviewed varied as to whether they used functional skills or GCSE programmes of study.⁷⁷ Several employers did, however, note a preference for functional skills due to them being *'more application-based, in terms of more about the kind of skills they'd need to know in the workplace'*.⁷⁸ The Richard Review also emphasises the importance of *'sufficiently functional'* maths training within apprenticeship programmes.⁷⁹

4.3 International Policy Lessons

▶ **The success of vocational maths policies is dictated by both the cultural importance given to maths and the success of pre-16 maths.**

Countries' vocational maths policies are inextricably linked to the cultural context which dictates the relative importance given to maths education, and the extent to which vocational learners have developed mathematical knowledge and understanding at a pre-vocational level. The extent to which learners gain maths skills while still in school has a particularly strong link to both engagement and outcomes in vocational maths. Advantages gained during pre-16 years are useful to learners as they move into the vocational stream. In Germany, the early divide between academic and vocational routes affects student expectations and their engagement with mathematics.⁸⁰

▶ **Greater equity, as well as quality, can achieve better outcomes among individuals at risk of disengagement.**

Equity is another important consideration, particularly in the context of this study with its focus on learners who may not previously have attained in or engaged with maths. Greater equity in results suggests a lower proportion of individuals who do not benefit from the educational system. Shanghai has had particular success in ensuring depth of success in mathematics across the whole student population, not simply among the most talented.⁸¹ The small gap between the highest and lowest performers and the importance given to equality as well as quality has been identified as a key factor behind the city's success by the Shanghai PISA centre.⁸²

The success of Shanghai's students in maths may appear counter-intuitive, as conditions in Chinese schools are in many ways not those considered ideal for learning by proponents of mainstream educational theory. Classrooms are crowded, classes are large, teachers are dominant and students tend to be passive, following a rote learning approach not usually thought to support effective learning in maths. Some observers have detected in this view a Western-centric

► **Factors such as long study hours can mask poor policies in comparator countries.**

In some instances, policies (and successes) run counter to mainstream educational thinking. In Korea, long study hours and an explicitly academic approach have contributed to its PISA success,⁸³ hiding the fact that vocational maths policies are relatively poorly developed compared to elsewhere.

► **Many international policies tend to focus on the supply side rather than the demand side of maths.**

There are, however, factors in common across countries; most countries reviewed tend to focus on the supply side rather than the demand side, leading the Australian Council of Learned Academies (ACOLA) - which reviewed international science, technology, engineering and maths policies - to make the point that *'little genuine effort is made to establish whether, and to what extent, these expected benefits of STEM are manifest in the economy'*.⁸⁴

► **Setting minimum maths standards for entry into vocational settings does not necessarily lead to better outcomes.**

In Canada, Québec attempted to disassociate VET from low-achieving students by introducing a minimum requirement for vocational courses, including a requirement that secondary IV maths had been passed.⁸⁵ This appears not to have translated, necessarily, to better outcomes, as learners often still lack the necessary maths skills to complete the course.⁸⁶

► **The embedding of effective practice requires time.**

One maths specialist reported that countries with successful maths teaching are those in which *'development has been out of the political cycle'*.⁸⁷ It was noted that education in China, for example, runs on a ten year cycle managed by a maths subject group.⁸⁸ It was suggested that in the UK, in contrast, there is little time for *'practice to become embedded'*.⁸⁹

The Freudenthal Institute at the University of Utrecht has developed a 'framework of reference' for mathematical literacy in upper secondary vocational education (MBO), which aims to allow the mathematical requirements of all 241 occupations covered by MBO training to be mapped on a single framework. The framework succeeded in making mathematics more visible and explicit within the competency requirements of vocational education in the Netherlands.

The framework designers identified the inclusion of clear examples from work situations, which helped users to identify the mathematics 'hidden' within specific tasks and work processes in the qualification files, as a key factor in the success of the framework. These facilitated communication between 'mathematical' and 'vocational' stakeholders and helped each to recognise the other. Carefully planned communications, and consistency in terms of the individuals involved in communicating the framework, helped ensure that non-mathematical kenniscentra staff were able to judge and value the mathematical content and levels in qualification files.

5. Pedagogy and Practice in Institutional Settings

Section five discusses the pedagogy and practice of maths teaching, reviewing the following areas: curriculum and content; diagnosis and assessment; approaches to engaging learners; pedagogical models and approaches; tools and resources; and ways in which maths is embedded in vocational contexts. Several successful pedagogical models from the Netherlands, Australia and Canada are reviewed in section 5.4 followed by useful resources and tools, such as technological support, in section 5.5. The importance of embedding maths in vocational contexts is highlighted at the end of the section along with useful approaches and tools for embedding maths in the workplace.

5.1 Curricula and Content

► **Countries tend to prioritise either international league tables (through abstract maths) or economic development (through functional maths).**

The content debate, in much of the literature reviewed, is around whether the preference should be a focus on abstract mathematics or functional mathematics, which is arguably similar to the distinction made above between maths and numeracy skills. Most studies point to the value of the latter for vocational learners. Obviously the exact form of the content depends, to a certain extent, on the vocational subject being studied; while there may be a core of topics that are critical for life and work, occupations also have specific requirements. Lab technicians, for example, need an in-depth understanding of statistics, including linear regression.⁹⁰

At an upper secondary level, countries tend to prioritise either maths concepts, focusing particularly on in-depth knowledge of curriculum content, which tends to be part of a policy agenda to gain momentum in

While many of the higher ranked PISA nations in Asia traditionally favour a conceptual maths approach, ACOLA report a recent *'shift in focus towards nurturing generic skills of creativity, problem solving, collaboration and higher order thinking'*, commonly adopted by the United States, Canada, the United Kingdom and Europe. In Singapore, this shift has corresponded to improved PISA rankings, while Japan reversed its move to introduce a more relaxed maths curriculum after falling PISA results.

the PISA/TIMSS rankings; or generic competencies, linked to a policy focus on maths skills for economic development.⁹¹ These priorities often shift within countries. According to ACOLA, many of the higher ranked PISA nations in Asia - which have tended to focus more on the conceptual side of maths - 'report a shift in focus towards nurturing generic skills of creativity, problem solving, collaboration and higher order thinking', the approach currently favoured by the United States, Canada, the United Kingdom and Europe.⁹² While it has been reported that Singapore gained its current PISA league table position by abandoning a traditional approach to teaching maths lessons and instead focusing on developing creativity,⁹³ Japan's move to introduce a more relaxed maths curriculum was reversed after falling PISA results.⁹⁴

▶ **Training ambitions influence both content and pedagogy.**

Content depends on the ambition of the training programme - is the maths element of any programme included so that learners can gain better maths skills, so that they are numerate, or so that they have maths skills appropriate for their employment? In Germany, maths education in VET is specifically aimed at consolidating learners' numeracy skills.⁹⁵ There is also a broader aspiration that all maths teaching should develop logical thinking, philosophical understanding and self-reflection.⁹⁶

There is also a direct relationship between curriculum ambition and pedagogy, as overarching ambition has an impact upon teaching and learning styles. In Ireland, for example, which has prioritised examination results, maths teaching is typically delivered using a didactic approach that focuses on rote learning specific mathematical procedures in order to achieve good examination results.⁹⁷ This approach to learning has been critiqued for offering limited scope for questioning, discovery and exploration in mathematics learning.⁹⁸

In Singapore, the school maths curriculum is based around five ideas linked to mathematical problem solving:⁹⁹

- **Attitudes:** beliefs, interest, appreciation, confidence and perseverance.
- **Metacognition:** monitoring one's own thinking and self-regulation of learning.
- **Processes:** reasoning, communication and connections; thinking skills and heuristics; and applications and modelling.
- **Concepts:** numerical, algebraic, geometrical, statistical, probabilistic and analytical.
- **Skills:** numerical calculation, algebraic manipulation, spatial visualisation, data analysis, measurement, use of mathematical tools and estimation.

▶ **Some countries have seen a move away from centrally and/or politically determined curricula.**

The extent to which maths curricula should be set centrally, or indeed by government itself, remains a matter of debate for policy makers and practitioners. Germany has attempted to find a balance by establishing a formal syllabus in maths that allows for a degree of local flexibility.¹⁰⁰ As noted above, China sets curricula according to ten year cycles; it is set by an expert team and is free from political interference.¹⁰¹

Countries in which teachers have been given a high degree of autonomy over how to interpret the curriculum have tended to experience good outcomes in maths.¹⁰² While there is a huge range of factors that influence these outcomes, it is likely that teacher autonomy - when paired with factors such as excellent teacher training, good and relevant curriculum content, etc - is likely to have a positive influence, as it enables teachers to tailor their teaching to the needs of different learners.

Effective Practice in Mathematics Education in the Vocational Classroom

A cross-national project (Northern Ireland, Denmark, Finland, Scotland and the Netherlands) was developed by six partner institutions to identify and share best practice in vocational maths, with the aims of sharing best practice and expertise in maths pedagogy; developing a short in-service course for teachers; exploring CPD and accreditation for teachers; and providing a resource toolkit. The project has produced, and made publicly available:

- An in-service teacher training course, including collaborative learning, contexts for learning, dialogic learning and problem-based learning.
- A resources toolkit, including ice-breakers, energisers, activities and teaching ideas.

The project, according to its own internal evaluation, has led to improved confidence and competence among practitioners, and an improved learning experience for students. Full information and resources are available at <https://sites.google.com/site/leovetmaths/>.

▶ **Applied maths may effectively combine basic maths skills and more advanced concepts.**

Study results vary as to the appropriate content required for successful outcomes in post-16 vocational maths. Some organisations have argued for a focus on numeracy and the basic skills needed for life and work,¹⁰³ while others promote a more conceptual approach to maths. A study by ACME, for example, found that teaching underlying concepts and abstract structures is critical, otherwise learners will not be able to apply their learning across a range of situations; a utilitarian approach also 'deprives them of the full opportunity for intellectual development which mathematics can provide and reduces the chance that they will appreciate and enjoy mathematics and wish to take it further'.¹⁰⁴ Applied maths might be a way to achieve elements from both approaches, as some argue that applied maths encompasses both numeracy/basic skills and more advanced concepts such as complex algebraic equations; equally, the proponents of a

focus on numeracy argue that complex concepts underline these approaches. Applied maths, as explored further below, appears to have particular resonance in vocational contexts. It is also, according to some, more useful to vocational learners' career development than specific academic qualifications.¹⁰⁵

► **While content should be tailored to individual needs wherever possible, there are restrictions on vocational practitioners in determining content.**

It is also important, where curricula allow, to tailor content to specific learners' needs. Disengaged learners, particularly those who have had negative learning experiences in the past, may need to be supported to develop an identity through which they can view themselves as being competent to deal with real-life situations, such as money, employment and the law; numeracy focusing on these topics could be a first step.¹⁰⁶ It should be noted, however, that vocational practitioners have relatively little freedom to determine content, as this is dictated by the qualifications towards which the curricula they teach are aimed. Within the constraints of nationally set curricula, teachers and trainers have more freedom over their teaching methods and approaches than they do over content; these are explored below.

5.2 Diagnosis and Assessment

Students in certain colleges may be assessed at the course outset and then placed on particular courses in line with their GCSE grades. One maths specialist noted, for example, that those students achieving a D grade at GCSE in maths would most likely be placed onto a BTEC level 2 course. Learners would then either undertake GCSE re-sits or continue maths learning as part of the course. Re-sits would, however, not be restricted to maths and ordinarily entail undertaking examinations in several subjects.

► **Post-16 teaching should be based on individual needs rather than focusing on the entire GCSE course.**

It was suggested that post-16 maths teaching should be based on the current level of maths that students have gained through 11 years of learning maths rather than teaching the whole GCSE course in its entirety. One maths specialist noted that students arrive on post-16 courses with prior knowledge: *'often when they come into FE a lot of people treat them as if they're blank slates and say we have to get through the whole curriculum, when actually we have to cover the misunderstandings and the areas where they don't have knowledge, and those are the things that they've got to address, not all the things that they do know'*. Another maths specialist commented that *'the biggest issue is that students who already know something, and they are trying to get themselves to a place where they have a qualification, someone needs to do some diagnostic work to find out what they do know about, what they don't know about and in what context'*.¹⁰⁷

'Often when [learners] come into FE, a lot of people treat them as if they're blank slates and say we have to get through the whole curriculum, when actually we have to cover the misunderstandings and the areas where they don't have knowledge.' - Interview with maths specialist

► **There is a need for accurate and informal assessment early in the course.**

It is imperative that learners' skills levels are accurately assessed at the commencement of the course.¹⁰⁸ Relying on previous formal assessments may give an incomplete picture of actual skills levels, particularly among learners who have struggled with maths and may have low levels of confidence in it.¹⁰⁹ While teachers need to understand learners' actual abilities at an early point within the course, it must also be considered that in some cases, previous negative experience of assessment may lead to failure or disengagement by the learner.¹¹⁰ Providing feedback to learners following assessment is also important. Good practice guidelines on initial assessment developed by the then Department for Education & Skills stated that *'learners often underestimate their skills, and positive feedback [following assessment] can enhance their self-confidence'*.¹¹¹

► **Diagnostic assessments can deliver more detailed learner profiles.**

Skills checks and assessments vary greatly depending on the institution in question. Organisations may, for example, incorporate skills checks with initial assessments and diagnostic assessments leading to detailed individual learning plans. Skills checks allow training providers to establish learners' individual maths skills levels through a short 10 minute test, and the initial assessment typically allows for courses at appropriate levels to be selected for learners. Both skills checks and initial assessments ordinarily comprise both an individual interview and short skills test. Diagnostic assessments provide a more detailed learner profile in order that individual learning plans can be written. These steps may be carried out well in advance of the course by training providers or indeed be carried out, for example, at job and recruitment centres.¹¹²

► **Pre-course testing can be inaccurate and unpopular with learners.**

Some models of assessment include testing prior to the commencement of courses. Pre-course testing can, however, be off-putting to learners. The accuracy of such testing has also been called into question. Teachers and trainers would therefore require high levels of diagnostic skills. It has been suggested that individualised interviews are perhaps a more positive way of determining learners' maths skills at the course outset. Individualised interviews would, inevitably, require dedicated teacher time for interviews at the course outset. Awareness sessions regarding the identification of learners' difficulties in maths learning have also been suggested as a useful approach for vocational trainers. It is noted, however, that *'where this strategy has been tried, there has been a significant lack of interest by staff'*.¹¹³

► **Learner engagement and provider focus is affected by the presence of maths in end of course assessments.**

Assessment at the end of the course also often dictates the extent to which maths is given sufficient attention by learners and curriculum time by education providers. This inevitably directly impacts learner engagement. In Germany, for example, the final assessment at the end of apprenticeships (the Chamber exam) does not take account of school performance, which includes maths.¹¹⁴ This can lead to apprentices focusing more on the vocational learning and less on developing and consolidating maths learning.¹¹⁵ In a multi-country study that includes Germany, vocational instructors expressed concern that while mathematical knowledge does not affect the ability to graduate successfully from VET programmes, it may limit the ability to function effectively in the workplace.¹¹⁶

5.3 Engaging Learners

It is widely accepted that many young adults on vocational courses can be disengaged from and have largely negative attitudes towards learning maths.¹¹⁷ Disengaged learners are often affected by multiple factors, including negative prior experiences with learning and personal issues at home affecting learning, compounded by low aspiration.¹¹⁸ One of the most frequent comments by teachers in a recent survey was that learners, having spent five years taking maths at secondary school and failing to achieve a passing grade, were demotivated and disillusioned. The lack of confidence that some suffered was made worse by the poor experience they had at school where, they claimed, their individual learning needs had not been considered. GCSE maths is seen as an academic subject that ‘switched off’ most of the learners.¹¹⁹ Several further studies point to the new maths GCSE requirement in FE having a potentially demotivating effect for students, especially given that for many students compulsory study may be closely associated with earlier school experiences.¹²⁰

► **Maladaptive beliefs are prevalent but mindsets can be changed quickly.**

Many young people, through their previous study of maths in school, have ‘maladaptive beliefs’ about learning maths; these can be characterised, according to one New Zealand study, as ‘those in which mathematics is characterised as facts, rules and procedures, thought to be learned primarily by rehearsal strategies, and perceived to reflect one’s intellectual abilities’.¹²¹ Maladaptive beliefs can contribute both to poor engagement and low achievement.¹²²

Extensive research has been conducted on challenging maladaptive beliefs about maths.¹²³ This includes work on changing mindsets and improving ‘mathematical resilience’.¹²⁴ Learners’ mindsets must be changed, according to one maths specialist, from ‘I can’t do maths’ to ‘I can’t do maths yet’.¹²⁵ A college representative noted the positive results achieved by tackling ‘negative psychology with a few minutes each session looking at attitudes to maths, growth mindsets, value of belief, [and] relating effort to success’.¹²⁶ One maths specialist commented that ‘in the early stages [of teaching] it’s 90% psychology and only 10% maths’ as many students’ perception of maths stems from negative association pre-GCSE.¹²⁷ It was suggested that young people should be encouraged to understand that they are able to gain maths qualifications.¹²⁸

One provider interviewed stressed the need for further support to address learners’ negative perceptions towards maths. The interviewee noted that ‘we have got to figure out that it is not throwing more funds and resources or vocationally relevant stuff, it is having to overcome deep fears and dislike, especially the fact that this is compulsory [...] it is not just behaviour it is managing fear, it is managing a very deep sense of failure, being rubbish’.¹²⁹

Growth Mindset

Carole Dweck’s work on mindset in learning has found that students with a fixed mindset (those who believed maths and science abilities are based on talent) performed less well than students with a growth mindset. While growth mindset students typically improved at a faster rate than fixed mindset students of comparable ability, the growth mindset students tended to focus more on learning rather than results (i.e. improved results were a by-product of students’ focus on learning). The attitude of teachers was also found to be important. Growth mindset teachers encourage students to believe that they can improve with hard work, while fixed mindset teachers reinforce students’ perception that either they are or are not ‘maths people’. Teachers can help support student learning not only by adopting an underlying growth mindset approach but also making the growth mindset approach explicit to students and providing them with up-to-date evidence that supports this view of learning. Praise should also focus more on effort than talent.

Teachers can promote a growth mindset in three key ways:

- ‘By teaching students about the new science of brain plasticity and the new view of talent and giftedness as dynamic attributes that can be developed.
- ‘Through the portrayal of challenges, effort, and mistakes as highly valued.
- ‘Through process praise and feedback.’

Dweck, C.S. (2008) Mindsets and Math/Science Achievement

► **There is need for a range of tools and strategies.**

Young people who have maladaptive beliefs about learning maths are more likely than other students to learn by rote, and need encouragement to undertake a broader approach to learning; this includes work by the practitioner to move away from explanations - as this group of learners can believe that passive listening is sufficient for learning - and

towards the incorporation of broader learning strategies.¹³⁰ These might include group work, getting learners to explain ideas and solutions to peers and teachers, and strategic problem solving.¹³¹

Other areas which may support or detract from propensity to engage with learning include an individual's socioeconomic status, ethnicity and the level of peer pressure they experience to behave in a certain way¹³². The reasons that learners disengage from learning are diverse, and therefore strategies to reengage them with learning must be equally so.¹³³ According to NCVER, 'there is growing appreciation of the deeply entrenched and complex nature of disadvantage and the difficulties associated with engaging and keeping individuals in skill development opportunities'.¹³⁴ As with tailoring maths specific content, with general disengagement it is essential to keep individual learner needs and priorities at the centre of any activity.¹³⁵ There is no one easy solution and teachers must be supported - through training and CPD - to have a range of tools available in order to ensure a range of available approaches.

► **Learners need a safe and organised environment, sufficient resources and a sense of community.**

The learning environment is key, including an ordered and safe physical environment, adequacy of resources and a sense of community. According to a study by Papalia, Feldman and Martorell on educational and vocational issues related to human development, education providers that support student achievement and minimise disengagement foster 'the belief that all students can learn. [They also offer] opportunities for extracurricular activities, which keep students engaged and prevent them from getting into trouble after school. Teachers trust, respect, and care about students and have high expectations for them as well as confidence in their own ability to help students succeed.'¹³⁶ Small teaching groups are ideal so that teachers can give greater individual support to students.¹³⁷

Group learning and one to one support were also found to be useful in supporting learner motivation.¹³⁸ Peer-learning is also an effective way to build confidence amongst learners, in addition to enhancing learner progression.¹³⁹ Learners in one study were provided with problem-solving exercises along with solutions to mathematical problems, and were found to adopt various different ways of thinking about mathematical problems by discussing exercises with each other.¹⁴⁰ Other successful peer-learning strategies have included 'competitive marking' amongst learners by their peers, competitive activities to engage learners, and group break-out sessions to discuss 'self-regulated learning strategies'.¹⁴¹

Classroom practice needs to change substantially in order to develop real-life maths skills in learners, moving away from the standard approach of explanation-example-exercises. In addition to incorporating real-world exercises which are of interest to groups of learners, important elements include:

- Not intervening immediately when learners struggle, but instead offering support when different approaches have been attempted and exhausted.
- Giving learners the responsibility to correct their own and others' work, rather than expecting the teacher to confirm answers.
- Tailoring supplementary questions to the progress of individual learners in order to support further development.

Bukhardt, H. (2008). Quantitative Literacy for All: How Can We Make it Happen. In Calculation vs. Context: Quantitative Literacy and Its Implications for Teacher Education. USA: The Mathematical Association of America.

One provider noted success in using a teaching assistant 'to follow up maths lessons by rotating each student in about half hour one to one slots to revisit current maths topics. This proved beneficial to tackle identified weaknesses.'¹⁴² Learners can also be motivated through progress tracking.¹⁴³ One provider noted that they 'have a checklist at the beginning of sessions to help [learners] define what they are going to cover; they record percentage scores on each block so they can see how they are doing'.¹⁴⁴ Positive incentive schemes were also found to work well, when funding is available, which reward students with a certificate or small prize for putting in a high level of effort (positive incentive schemes are also found to work well with attainment,¹⁴⁵ although likely to have a greater impact if focused on effort).

► **Supporting learners to build their self esteem is critical to improving outcomes.**

Learners with higher self-esteem are more likely to achieve well at school and college.¹⁴⁶ Vocational learners tend to have lower self-esteem than academic learners (and this effect is increased where academic and vocational learners study at the same institution).¹⁴⁷ Raising self-esteem is therefore critical to ensuring better outcomes both in general, and in maths, where self-esteem among those who have not achieved well previously may be especially low.¹⁴⁸ Learners who are motivated to learn maths are more likely to do well in it.¹⁴⁹ The relationship between the teacher and learner is absolutely vital to the successful delivery of vocational maths programmes.¹⁵⁰ High expectations of their students by teachers can lead to positive reinforcement whereby students have high expectations both of themselves and their teachers.¹⁵¹ According to one Australian study, disengaged learners particularly benefited from supportive, respectful and equal relationships between peers and tutors.¹⁵²

Three providers interviewed also commented on the importance of building rapport between learners and tutors. It was also noted that 'reciprocity is important - [you] need to establish mutual trust and understanding'. Tutors in one college were said to be called by their first names as opposed to 'sir' as a way of building rapport. One further provider commented that they try to establish a good relationship with each student by meeting with them individually before the course begins.

According to one college, there should be greater focus on areas for improvement and support for learners than grades: 'hard work and persistence was praised, not necessarily high grades. When questions were raised regarding grade

boundaries we would just tell students they would need to do as well as they possibly could.¹⁵³ This approach has led, according to the college representative, to a substantial increase in GCSE pass rates.¹⁵⁴

► **Learner motivation is affected by the perception of relevance.**

Learners' motivations can also be influenced by the extent to which they find maths curricula to be relevant; this can be affected by the perceived value of the qualification, the extent to which learners find it practically useful and the extent to which they are able to learn transferable skills.¹⁵⁵ Self-esteem was also found to be affected by directly addressing and working on concerns that learners have regarding real-life situations.¹⁵⁶ Framing questions in 'an adult context' and teaching maths using real-life situations was said to be far more engaging for learners.¹⁵⁷

Gaining appropriate careers advice and guidance is important in helping learners to develop a learning pathway and to raise aspirations.¹⁵⁸ Relating maths to future career options was further found to be a motivating factor.¹⁵⁹ One provider, for example, commented that they are 'getting employers in and we are trying to get more employers in, but until the employer really says how relevant maths is for career progression, then sometime it falls a little bit on deaf ears to our students'.¹⁶⁰

5.4 Pedagogical Models and Approaches

One of the key findings to come out of this research is that no single approach is appropriate for learners; approaches must be combined and tailored according to the specific needs of the learners being taught. Formal models and approaches used include the following:

Reality, Abstraction, Mathematics and Reflection (RAMR) Model

The RAMR model has been used widely in mathematics education in Australia, increasingly so in vocational settings. It has been a key part of the YuMi Deadly Maths Program, which has been running in over 100 Queensland schools.¹⁶¹ Students' current knowledge, linked to their social and cultural backgrounds, underpins their learning of maths. Learners then generate their own symbols 'to represent their understanding of the mathematical process. These symbol systems are then compared to, and assist in understanding, the meanings of standard symbols, symbolic language and their connection to reality.'¹⁶² This process enables students to explore the underlying structure of maths. Problem solving and creativity are both key to this model.¹⁶³

Discovery Math

Discovery Math is a Canadian approach that encourages learners to apply problem solving skills to mathematical problems, rather than utilising traditional memorisation and rote learning techniques: 'the more problem-solving that you've done whether it's in math class or in science class or in one's job in the real world, the better able you are to solve the next problem that comes along'.¹⁶⁴ The programme includes learning principles such as the 'discovery of basic facts through concrete experiences; using language to think about those experiences; using mental imagery to further internalise and understand basic math experiences, operations and principles; and integrating and applying the understanding to problem solving'.¹⁶⁵

Realistic Maths Education (RME)

The Netherlands is the birthplace of Realistic Mathematics Education (RME), an approach to teaching maths - since adopted widely elsewhere, including the UK - that emphasises the use of real-world examples, activities that encourage pupils to move from informal to formal representations, and 'making sense' rather than learning formal ideas in the early stages of maths.¹⁶⁶ RME has been praised by the European Commission as one solution to existing preconceptions of maths as difficult, abstract, boring and irrelevant to real life, by organising lessons around interdisciplinary themes and making explicit connections to other subject areas.¹⁶⁷ RME developed as a reaction against the prevailing 'mechanistic' approach to mathematics teaching in the Netherlands in the early 1970s, emphasising social relevance and mathematics as a human activity rather than a subject matter to be transmitted.¹⁶⁸ In educational contexts, this puts the emphasis on activity - on giving students guided opportunities to 're-invent' mathematics by doing it.¹⁶⁹ Problems are presented in a contextualised fashion throughout the learning process, in contrast to traditional approaches which only use context as a way to illustrate what has been learned abstractly.¹⁷⁰ In RME, working on contextualised problems is used to develop learners' mathematical tools and understanding; these can then be used to solve related problems and eventually to form the basis of more formal mathematical knowledge.¹⁷¹ RME was strongly influenced by socio-constructivist theories of education, seeing the student as an active participant in the learning process, allowing them to develop mathematical insights and tools rather than passively receiving information.¹⁷²

Cultural and linguistic pedagogical issues present particular challenges with teaching mathematics in vocational settings in the Netherlands, since learners from minority communities (mainly Turkish and Moroccan) are over-represented in the vocational stream.¹⁷³ These learners may lack the foundation in abstract language skills needed to build success in mathematics given that the existing RME-based curriculum has a tendency to rely on examples that are highly dependent on Dutch culture.¹⁷⁴ These learners may also be more culturally familiar with a rote-learning and memorisation approach to education that does not sit well with the dominant Realistic Mathematics Education pedagogical approach.¹⁷⁵

Dialogic Teaching and Learning

Literature on the dialogic approach to teaching maths tends to be focused on school-level rather than vocational maths. Some studies, however, make clear that it is useful in vocational settings (there are, for example, resources available on dialogic teaching on the ‘Best Practice in Mathematics Education in the Vocational Classroom’ toolkit outlined above)¹⁷⁶. Dialogic teaching is simply the incorporation of talk in the development of mathematical concepts.¹⁷⁷ It is beyond the scope of this paper to go into details on this approach, as it is clear that practitioners need an in-depth knowledge and understanding of it in order to incorporate it successfully into maths teaching; one example of a practitioner booklet is available [here](#).

Metacognition

The use of innovative ways to develop thinking skills and metacognition in mathematical learning have been tested in several studies.¹⁷⁸ One study, for example, introduced a distinct mathematical language to encourage college learners to engage with mathematical concepts and reform problem-solving questions into understandable language.¹⁷⁹ Learners used real-life examples of maths to form their own level 2 numeracy questions. Following further explanation from practitioners, these questions were then reformed into a mathematical language called ‘Mathslish’.¹⁸⁰ The study reported that ‘over 70% of learners gained a greater understanding of maths in everyday life, and over 70% of learners felt more confident in their mathematical ability’.¹⁸¹

5.5 Tools and Resources

UK Initiatives

Initiatives aimed at improving maths skills in the post-16 vocational sector include the Maths Fest, organised by National Numeracy in the Yorkshire and the Humber region. The event attracts teachers at multiple levels and young people aged 16-19. National numeracy has run a further maths enhancement programme and ‘close communities’, managed by CTN, for those involved in the delivery of GCSEs. The maths specialist interviewed noted the importance of building communities as ‘there were people who came to those who were perhaps sent by their employers, but who otherwise wouldn’t have engaged in something like that’ and suggested that more events like these should be held.¹⁸²

Technology

Many studies demonstrate the effectiveness of utilising technology to support learner progression in sixth form, further education, higher education and vocational settings.¹⁸³ Current research has espoused the benefits of online resources while at the same highlighting the importance of ensuring that online material is regularly updated and that links are live.¹⁸⁴ Providers have also reported success in employing social media and other online platforms in maths teaching.¹⁸⁵ Where the possibility does exist to utilise online learning platforms, maths programmes can be tailored to each individual sector thus supporting trainers.¹⁸⁶ Programmes can additionally be personalised for individual learners, offering them greater flexibility and learning tailored to their needs.

Key technology formats include video, Interactive Screen Experiments (ISEs), Virtual Learning Environments (VLEs) and software such as GeoGebra.¹⁸⁷ VLEs engage learners through multiple virtual platforms, providing guidance through curriculum subjects via various websites, podcasts and online resources.¹⁸⁸ VLEs also allow for an highly interactive, engaging environment for learners who are able to upload their own content to sites.¹⁸⁹ Success in numeracy teaching has also been achieved by engaging learners with platforms such as Mymaths and Whiteboardmaths.¹⁹⁰ Technology successfully employed by learndirect includes the use of online interactive whiteboards; instant messaging; online communities; mobile applications and SMS; video libraries; virtual classrooms; WebEx; Webchat; and YouTube.¹⁹¹ One study showed the positive results achieved in assisting learners to understand mathematical concepts through the use of Makaton - which utilises speech, signs and symbols to support learning.¹⁹²

Many providers reported using a wide range of resources to engage maths learners. These include the use of electronic white boards; virtual learning environments; games including interactive games; visual and practical exercises; Standards Unit Box; BKSB; Maths Watch DVD; Edexcel revision guides; BBC Skillswise; Kangaroo Maths; My Maths; Transom starter for the day; Online Daydream posters; Maths Watch; Exam Solutions video site; Hegarty Maths website; m4maths website; Skills Workshop website; Manchester University’s Making Sense of Mathematics; and Autograph Maths online. Several providers also developed their own resources using, for example, topical data from the Olympics, the World Cup and on migration.

Self-Paced Training Materials

While self-paced training materials may work well in helping disengaged learners to build a positive relationship with vocational maths,¹⁹³ they may not be appropriate for learners with low levels of literacy and numeracy.¹⁹⁴ This is partly due to such materials requiring a relatively high level of literacy, and partly due to a need to be assertive when requiring assistance from the teacher or tutor, especially for learners who may have low levels of self-confidence.¹⁹⁵ In contrast, another study has suggested that engaged or ‘indifferent’ learners benefit less than disengaged learners from self-paced materials as they need to be stretched towards concepts and strategies.¹⁹⁶

Other projects, however, have seen success in the use of self-paced training materials. The Tallaght Probation Project Service Provision in Ireland - although it targets young adults (18-35 years of age) within the criminal justice system, which is slightly out of scope for this study - offers a good example of innovative maths learning through the use of the ‘writeon’ software package.¹⁹⁷ This software allows individuals to develop numeracy and maths skills at their own pace,

using individual assessments and planning to support independent learning through carefully structured activities.¹⁹⁸ Tutors also work closely with learners to address any emotional barriers to maths learning; for example, each learner discusses their experience of formal schooling in ‘*deep down detail*’.¹⁹⁹ The use of ‘writeon’ has been effective in enabling learners to understand their learning needs²⁰⁰. Most learners move quickly through levels 1-2 on Ireland’s National Qualifications Framework; some, but not all, learners move on to level 3.²⁰¹ Success at working with ‘writeon’ also enables learners to consider further formal assessment and accreditation.²⁰²

5.6 Embedding Approaches in Vocational Contexts

Learning by doing, or kinaesthetic learning (for example, by embedding maths into vocational and real-life contexts) has been found to be highly valuable. The use of learning by doing was identified as valuable in increasing the effectiveness of learning, and supporting the acquisition and retention of numeracy skills.²⁰³ Learning by doing methods can encourage students to do maths without realising they are doing it.²⁰⁴ Embedding maths teaching into vocational training has also been shown to be highly successful in those cases where trainees are unable to undertake separate maths courses²⁰⁵ and is also valuable for reinforcing maths skills and highlighting the relevance of maths. Contextualising maths is a proven way in which vocational students can be taught. Methods can include giving students relevant problems to solve and relevant contexts that enable them to understand abstract maths or concepts.²⁰⁶ Studies, while not entirely rigorous, do suggest significant improvements in outcomes.

Students are generally given relevant problems to solve in real-life contexts that enable them to understand abstract maths or concepts.²⁰⁷ One functional maths tutor, in an example drawn from a recent study, offered an effective solution to embedding maths learning across a range of disciplines by focusing on calculations related to broader real-life scenarios: alcohol consumption, calories and the cost of smoking. The emphasis was on engaging learners and discussion around the scenarios was allowed to deviate from the maths that was being taught.²⁰⁸ According to the research report which captured this element of practice: *‘the contexts were convincing for them and they readily described situations in their own lives which required mathematics’*.²⁰⁹ Effective practices include the following:²¹⁰

- **Vocational or real-life contexts should be authentic** and not feel contrived. At the same time, subjects of general interest can be useful to learners, as it enables broad-ranging discussion and engagement which might not be possible with a focus on purely vocational contexts.
- When using discussions such as these in which to embed maths, **the maths itself should not be prioritised at the expense of the discussions**, or teachers/tutors risk losing the engagement of the class - which defeats the end point.
- Situating maths in real-life situations in which learners are interested can have knock-on effects in their **recognition of potential relevance to their vocational area of study and/or their daily lives**.
- **Greater engagement can engender greater levels of confidence** among learners.

Relating maths problems directly to vocations was also found to be successful in engaging learners. Several maths specialists stressed the importance of embedding approaches to maths teaching in vocational and real-life contexts. It was noted that *‘when they’re [learners] [are] being asked to do completely stupid things, that people don’t actually do in real life, then it makes everybody look very, very silly’*. One maths specialist felt that the importance and reality of using maths in everyday contexts *‘needs to be drawn out more clearly’*, as many vocational tutors use maths every day for training in such courses as bricklaying and plumbing. It was seen as highly important that vocational tutors in FE colleges understand maths and numeracy and are able to integrate learning into courses.²¹¹

► Contextualised learning improves understanding and information retention.

A variety of literature supports the idea that contextualised learning of maths improves both learners’ overall understanding and the extent to which they retain the information they have learned.²¹² It can be challenging, however, to implement in the face of wider curricula or competency-based training systems. In Australia, for example, topics can often be compartmentalised or atomised when aligned with the broader competency-based training framework, making contextualisation difficult to align.²¹³ Academics have also argued that *‘random insertion of contexts into assessment questions and classroom examples in an attempt to reflect real-life demand and to make mathematics more motivating and interesting... ignores the complexity, range and degree of students[’] experiences as well as the intricate relationships between an individual’s previous experience, mathematical goals and beliefs’*.²¹⁴

There is a body of academic thought suggesting that contextualising learning can reduce rigour and narrow learners’ understanding of maths.²¹⁵ It is therefore important to ensure that students are encouraged to apply contextualised mathematical tools across various areas in life, which reduces the risk that learning cannot be applied elsewhere. One provider found, for example, that *‘motor mechanic students who were wonderful with ratios and various bits and pieces when it related to cars’* were unable to apply the same maths to recipes *‘because they didn’t realise that what they were doing with the car had the same structure as what [they] were doing in the recipe’*.²¹⁶

► Embedding maths into vocational courses may reduce learner drop-out rates.

One study, by Casey et al, suggests that learners were less likely to drop out of courses when maths teaching was embedded into vocational learning at level 2.²¹⁷ For the sample chosen, there were up to 20% higher achievement rates when comparing fully embedded approaches with those that were not embedded.²¹⁸ Questionnaires delivered as part

of the above study, revealed some important findings about what makes embedding successful. Embedding works best when:²¹⁹

- The college does not give responsibility to an individual teacher for vocational learning and maths and English unless they are high skilled and qualified in both parts (which is rare).
- Vocational and maths teachers are working together to embed subjects increases the probability of learners to achieve.
- Formal shared planning between both teachers is undertaken.
- Departmental and institutional management structures support embodied provision in practice.
- Maths and English teachers are viewed by staff and learners as contributing to vocational aspirations; they understand and engage with the vocational area as part of their work; and are willing to develop their skills with regard to the vocational area of their work.
- An initial diagnostic assessment is used to contribute to the integration of LLN into vocational teaching.

▶ **New technology has supported the development of tools for embedding maths in vocational and real-life contexts, although this has been challenged by the pace of curriculum changes.**

New technologies have supported the development of tools which can help maths to be embedded into real world situations through interactive environments. Online resources are provided by a range of organisations, and are both free and paid for. A 2009 study found online maths resources to be seriously underdeveloped and to offer inadequate curriculum coverage;²²⁰ while web resources have developed rapidly in the interceding years, the swiftness and regularity of curriculum changes make it likely that maths resources have failed to keep pace.

Web based resources have a lot of potential to engage learners; most of the current generation of 16 to 19 year olds are 'e-mature'. A study by researchers at Leeds Metropolitan University found that *'allowing students to work at their own pace is a considerable advantage; however, to maintain interest the options and activities available need to be extensive. For those students that need more guidance to understand the maths, help can be provided in a number of different ways in an interactive environment. Text, drawings, animation, audio and video can be drawn on to provide supporting information to help students to understand.'*²²¹

6. Pedagogy and Practice in Employment Settings

The following sections reviews pedagogy and practice in employment settings both in the UK and international context. Findings highlight the need for employers to understand the individual maths competency levels of trainees and apprentices and focus on building confidence through practical teaching methods. One to one support and mentoring are discussed as useful approaches for engaging those who have previously struggled with maths, as well as ensuring that learners have ownership of the learning process. Functional maths skills are also noted as potentially more relevant for workplace settings.

6.1 UK Context

▶ **Key challenges for teaching young people maths in the workplace include disengaged learners, low pre-existing numeracy levels and a lack of appropriately qualified trainers.**

Employers interviewed noted a number of challenges in the teaching of maths to 16-19 year olds in the workplace. The key challenges reported were as follows²²²:

- Disengaged learners, whose negative previous experiences of studying maths in school were noted as a distinct challenge by several employers interviewed. Learners may also be disengaged as they fail to understand the relevance of maths for their chosen vocational path.
- Low levels of numeracy. Several employers noted that many young people struggle with very basic numeracy.
- A lack of trainers qualified to teach maths in the workplace. Several employers noted that they themselves may not have undertaken maths training in many years.
- Resources for maths teaching at the appropriate level were found to be lacking in some workplaces.
- The relevance of GCSEs was called into question by employers interviewed.
- Limited timeframes to ensure that learners are at the required level to pass GCSE maths.
- A lack of employer engagement in standard setting and curriculum content.

▶ **Articulating the benefits of maths to employers can improve outcomes.**

Maths can be successfully integrated with on-the-job training by articulating the benefits of this approach to employers, such as fewer learner mistakes and a reduced need for supervision; by ensuring that the employer knows the maths qualification, including the level of maths it requires, towards which the learner is studying, and has support for methods and approaches to teaching the learner in the workplace; and ensuring that learners understand the relevance of maths to their specific workplace.²²³

► **Implementation of maths training in the workplace is more likely to be successful where it is integrated into wider company strategies.**

It is also important to integrate maths teaching into wider company strategies.²²⁴ Referring to Wolf (2005), a report from the Institute of Education notes that *'the success of workplace literacy and numeracy is in part likely to depend upon the extent to and ways in which the training is located within the organisation's wider learning and training development strategy and overall business strategy'*.²²⁵

6.2 International Lessons

The creation of realistic and appropriate work contexts in pre-employment training can help to prepare young people for the later use of maths in the workplace. A Danish study on vocational maths categorised the key differences between numeracy at work and traditional maths instruction at school:²²⁶

Numeracy at Work	Maths in School
All numbers have units of measurement or refer to something concrete.	The numbers often appear as pure numerical quantities.
Numbers and tasks have to be constructed.	Numbers and tasks are given.
A task often has different solutions.	A task only has one correct solution.
Accuracy is defined by the situation. Right/wrong is negotiable.	Accuracy is defined by the teacher. Right/wrong is not negotiable.
Solving tasks is done through collaboration.	Solving tasks is an individual matter, i.e. competition.
Tasks are full of 'noise'. The numbers are often 'dirty'.	Tasks are cleared of 'noise'. The numbers are 'clean'.
Reality requires the use of mathematical ideas and techniques.	Reality is a pretext to use mathematical ideas and techniques.
Solving tasks has practical consequences.	Solving tasks has no practical consequences.
Working tasks are defined and structured by the technology.	Mathematical tasks structure the teaching.

With this in mind, implications for pedagogy and practice, in terms of creating a more realistic work context for maths learning, might include:

- Giving learners the opportunity to construct their own tasks around work scenarios.
- Giving learners the opportunity to work together to solve tasks.
- Using real-life work scenarios and contexts wherever possible.
- Linked to this, ensuring that the work scenarios and contexts used allow for real units of measurement/concrete ideas to be used; and not teaching maths in abstract.
- Using occupation-specific technology as a basis for maths tasks.
- Giving learners practice in working on tasks for which there may be different right answers.

One German technology company has created an Action Day for Girls, focusing on female students who may consider maths or science training in a vocational context. This initiative responds directly to demographic changes that have led to a shortage in the labour supply, and addresses the fact that females are less likely to specialise in maths and science. 200 female students attended an action day for girls at this large company in Hamburg. Students were welcomed and given an information briefing by a business director and information to take away. There were then opportunities to talk to female employees in a range of roles and to discuss career opportunities which would use maths and science skills. This was followed by tours in small groups. This initiative illustrates the recognition by German industry of the need to engage more young people in mathematics, and in particular the role of positive action initiatives to engage a wider pool of young people in maths and science.

<http://www.spiegel.de/international/germany/germany-boosts-efforts-to-get-women-in-stem-fields-a-924023.html>

6.3 Training Requirements

► **The majority of employers do not have set requirements for maths skills for apprentices.**

The majority of employers interviewed do not have any set requirement for maths skills for apprentices aged 16-19. This was particularly seen to be the case in certain sectors, such as the hospitality sector. One employer, for example, reported that *'in hospitality it's more about the character of the person and then we can train in skills for the needs of the business - it's so much more about personality'*. Some employers do, however, request that learners have reached a certain level in maths, such as level 2 or 3 or even GCSE grade C. Certain posts also require higher maths qualifications for progression. An Army representative noted that GCSEs are only required for particular trades.²²⁷ Another employer also reported that it is common that half of all apprentices will not have achieved GCSE grade C in maths. A

representative from the Army further noted that learners are *'basically not good in the areas of fractions, decimals, percentages, both median range and, I find a bit odd, they're not very good at area and volume'*. Another employer stressed that building confidence is key for apprentices.²²⁸

▶ **The training requirements of young people in maths are heavily dependent on context.**

The learning needs of 16-19 year olds in vocational training are inevitably guided by the context in which the company works, though many employers interviewed noted that trainees would only be required to undertake very basic mathematical calculations.²²⁹ Maths knowledge needed for the food industry, for example, was said to be limited to very basic counting of stock, adding and subtraction.²³⁰ More advanced maths is, however, needed in certain professions.²³¹ Several employers noted that there are no specific requirements at all, given that many functions are now automated.²³² Specific maths skills were said to be needed mainly for those roles that would require someone with a degree or indeed for progression to more senior roles.²³³

▶ **Employers must work to understand the actual maths competency level of young people, rather than assuming a body of knowledge.**

It was found to be essential to understand the individual learner's competency level.²³⁴ One interviewee noted that *'the challenge is ensuring we don't make assumptions of knowledge [and] we don't over-assume that ability to add up a column of numbers or do straight forward calculations in the head. It isn't necessarily there.'*²³⁵ Several employers interviewed use numeracy and aptitude tests to gauge the apprentice's level at the outset. An employer in the banking sector, for example, reported that they would use a diagnostic test to assess whether the apprentice's level of maths competency met the required standard of level 2. It was suggested that maths teaching be delayed for those apprentices who had previously struggled with maths.²³⁶ One employer in the construction trade reported that *'if we find there's one or two that just can't pass the tests [...] we put them as a labourer; we find that the boys often are late developers; they could only come into their own at the age of 18, 19, 20. So, they do the labourer, the practical stuff, first and then come onto the maths.'*²³⁷

6.4 Practice

▶ **Building confidence in apprentices is important to employers.**

One employer mentioned that they had introduced a 15 week pre-apprenticeship programme in order to increase confidence amongst learners. Another interviewee reported that they use easily comprehensible exercises to integrate maths learning into activities in order to build confidence: *'young people need to improve confidence in maths but we use settings like measuring the area of a car park or football pitch so they realise that they can do maths.'*²³⁸ Ownership of the learning process by apprentices was said to be very useful in engaging learners. One employer interviewed, for example, noted that, in contrast to classroom based learning dictated by a teacher, on-the-job training ensures that learners *'are actually applying knowledge and learning from a real situation. So they're given the opportunity to do it themselves, they're getting a bit of responsibility, a bit of ownership and I do think that's a good way to engage learners into something new.'*²³⁹

Learners were also said to be motivated to succeed in maths purely by being in a working environment. One employer interviewed noted that apprentices have *'got a passion to work on big commercial aircraft basically [...] we find that out during the interview process and just by bringing them to the hangars at Heathrow and Gatwick and showing them a big jumbo jet for the very first time you can see in their eyes that they want to be here'*. Another employer noted that *'young people need to work with people who need to do the same thing. They need to feel confident in an environment that doesn't feel too much like school.'* Progression dependent on passing maths tests was also said to be a motivating factor. One interviewee noted *'we find that there is quite a lot of encouragement from the older guys who've been there and done it, so as they get promoted they'll encourage the others to follow in their footsteps, if you like. They see the advantages, and it's perfectly black and white, if you have these qualifications you can be promoted, if you don't then you can't.'*²⁴⁰

▶ **Some apprentices are provided with one to one support and mentoring from employers.**

Many employers noted that flexibility and one to one support is given to those apprentices who may be struggling with maths elements, where possible and dependent upon class size and tutor availability. An employer from the Navy, for example, noted that *'the class sizes that we get are usually relatively small, so we will focus into the individual, rather than have it a blanket, one size fits all solution'*. An employer in the hospitality sector reported that one to one support can additionally be given by telephone as well as face to face. It was noted that one to one focus *'works quite well as there is a crew trainer with them in the restaurant on a day to day basis so they form a good relationship. A 1:1 relationship with the same trainer throughout their training. [It's] expensive in terms of resources but great benefit for business including increasing confidence - they can deal with situations better and use their own initiatives, this is good for business and we have better KPIs.'* Another employer noted that *'the majority of our delivery is one to one which benefits people hugely with all those questions they never wanted to ask in a classroom of 30 - they don't mind asking when it's one to one. That can really boost people's confidence when they get that understanding rather than a one-way [approach] of delivering it.'*²⁴¹ Individualised support further allows for confidence building amongst those learners who had previously struggled with maths.²⁴²

One employer noted that they utilise both one to one and small group approaches to training, focusing on issues such as financial understanding, managing budgets and business planning. Topics of learning were said to be *'very much focused around... business needs'*. Other employers reported that they individualise learning plans to apprentices and sometimes use mentors or a *'buddy system'* to support the learner. One interviewee noted that *'the mentor shows them exactly what to do and shows them the basic maths and English as they go along because they've got to look at the maths if they're reading*

instructions and looking at plans; they're having to look at drawings with measurements on and interpret that'. Another noted that they 'use like a buddy system, so regardless of what training aspect I'm doing, I'm always with them all the way. You're not just sort of thrown into it and there you go, there's a piece of paper, there's a test or there's this, just go and do it.' Diaries and workbooks containing maths exercises may also be used. It was further suggested that email support from tutors is important as young people aged 16-19 prefer using forms of social media rather than having phone calls with tutors.²⁴³ It was also suggested that maths software packages may be useful for employers to enable apprentices to continue with one to one learning.²⁴⁴

► **Practical teaching methods are considered vital to those delivering in-house training.**

While several employers reported outsourcing training to learning providers, many of those delivering training in-house emphasised the importance of practical teaching methods. One employer commented that 'when they [apprentices] start doing practical skills, their maths comes on leaps and bounds purely because when they're measuring a radiator and they can see why they're measuring a radiator – because it's got to go on a certain part of the wall. So, we don't really have problems in that respect, when it becomes a practical exercise. When it's pen and paper, we have huge problems.'²⁴⁵

The importance of using practical examples to relate maths to working problems was also noted, as many apprentices who have previously been low achieving in maths will struggle to understand written mathematical questions: 'unfortunately, the people that have got very low maths/English have also got very low social skills; they have been the ones that have been disaffected, and I think the problem we have as an industry is they seem to be tarred with a brush at school; that they're not achieving'. One employer also commented on the importance of showing apprentices why they need to use maths in their roles and adapting maths terminology to counter negative associations.²⁴⁶

Games and quizzes were, finally, found to be useful. One employer reported that they 'don't use a theory based maths approach' but rather work 'in a fun way' using quizzes and games such as snakes and ladders in maths teaching. An employer in the retail sector also noted that it's useful to 'make it sort of more like puzzles and challenges, so how somebody wants to solve something, so you try to make it something exciting that you need to solve or you make it go down the route of it's actually a practical [thing] - you need to understand the practical use of this and how it's used in a business and why it's important in a business'.²⁴⁷

► **Learner confidence and motivation can be improved by being in a working environment, ownership (by learners) of the learning process, good rapport with teachers and peers, and additional support.**

Learners were also said to be motivated to succeed in maths purely from being in a working environment.²⁴⁸ Confidence was said to be improved through the development of rapport between trainers and learners.²⁴⁹ Ownership of the learning process by apprentices was said to be very useful in engaging learners.²⁵⁰ Peer encouragement was also said to be a motivating factor.²⁵¹ One employer mentioned that they had introduced a 15 week pre-apprenticeship programme in order to increase confidence amongst learners.²⁵² Another interviewee reported using easily comprehensible exercises to integrate maths learning into activities in order to build confidence: 'young people need to improve confidence in maths but we use settings like measuring the area of a car park or football pitch so they realise that they can do maths'.²⁵³

► **Additional support given to apprentices may include tailored, individualised study programmes.**

Many employers noted that flexibility and one to one support are given to those apprentices who may be struggling with maths elements where possible and dependent upon class size and tutor availability.²⁵⁴ One to one support also allows programmes of study to be tailored and individualised.²⁵⁵ Individualised support further allows for confidence building amongst those learners who had previously struggled with maths.²⁵⁶

7. Professional Skills and Training

Section seven reviews the teaching context and notes the lack of skilled teachers and trainers in vocational maths settings, as well as the lack of confidence amongst training providers in vocational settings. Findings show that collaborative working supports learner progression and acts as a motivating factor for teaching staff. Galvanising student motivation should also be addressed through teacher training.

► **Skilled teachers and trainers in vocational maths settings are lacking.**

Many experts and providers interviewed suggested that the lack of qualified teaching staff may be responsible for the discontinuation of maths learning at the post-16 level.²⁵⁷ It was said that 'there are problems with trained teachers being able to deliver maths numeracy for that cohort; you can see that in lots of colleges - they struggle to find staff'.²⁵⁸ A provider interviewed noted that 'we, obviously, like every other college in the country, are suffering from the difficulty in getting innovative, experienced and suitably qualified maths teachers on board'.²⁵⁹ Another reported that the 'main challenge is the competence levels of our staff and their challenges in delivering maths as well because they all have barriers to maths and they're not happy with delivering that, especially when we changed to functional skills'. In addition to competency levels, knowledge of vocational areas can be a challenge in certain colleges. One college explained that they 'have specialist maths knowledge, but we don't necessarily have the knowledge of the vocational areas'.²⁶⁰

There were further said to be insufficient numbers of teachers qualified to teach core maths.²⁶¹ The current number of qualified maths teachers was found to be lacking; 2,500 maths teachers are needed by 2020 to teach core maths. It was

suggested that additional funding is needed for appropriate training as well as more time for teachers to focus on training.²⁶²

▶ **Lack of training and confidence are challenges for maths teachers in vocational settings.**

The importance of teachers to outcomes in post-16 maths is paramount. Studies suggest, however, that lower levels of pay in the further education and training sector in comparison to schools and sixth forms present a significant barrier to recruiting qualified teachers for GCSE maths. Many newly qualified teachers have also expressed the belief that FE students are less able and motivated.²⁶³ The same study shows that the motivation levels of the staff in FE colleges are worryingly low. A majority were not convinced that delivering GCSE maths to FE college students is worthwhile - so they have little desire to undertake an additional qualification to teach it.²⁶⁴

▶ **The availability and quality of maths-specific CPD and resources is inconsistent.**

Professional development offered by providers was found to vary according to institution. Several colleges offer various professional development streams, including the NCETM maths enhancement programme, the EMCETT (MEP) course and in-house training. Two college representatives noted that CPD tends to be 'generic' and offers little maths-specific professional development.²⁶⁵ Given the new requirements, lecturers in vocational settings need access to GCSE maths CPD in particular.²⁶⁶ Further CPD practices utilised by teachers include mentoring, e-learning and coaching.²⁶⁷ The NCETM has also worked with NANAMIC to provide CPD for teachers in FE colleges.²⁶⁸ Training is given to teachers who have mainly worked in functional skills to provide them with the skills for teaching GCSE maths.²⁶⁹ The ETF has additionally worked with maths specialists to develop FE and skills maths hubs.²⁷⁰

One college has introduced the 'Teaching and Learning Takeaway' so that tutors receive various new teaching resources and activities to undertake with learners. The Maths Fest was further found to be useful in connecting tutors to developments in maths teaching. One provider, for example, noted that they've 'become very isolated as a training provider and it's nice to actually meet up with people who are delivering the same qualification and do have learners that have challenges where maths are concerned and it's nice to discuss those and come up with resources and tactics of dealing with that'. One college had also formed 'maths hubs' that work with multiple schools to share best practice in maths teaching.²⁷¹

▶ **Teacher professional development may be limited by a lack of confidence in maths.**

According to an evidence review conducted by the The Commission on Adult Vocational Teaching and Learning (CAVTL), 'many of the vocational teachers and trainers we have spoken to, including those in the workplace, say they do not have the skills they know are required to teach literacy and numeracy, and they want to spend the precious time they have focusing on their vocational specialism'.²⁷² The Maths Enhancement Programme was intended to address some of these skills gaps²⁷³ and bursaries of up to £10,000 were available to attract graduates to train as teachers of maths (and other subjects) in FE.²⁷⁴ The Premium Graduate Initial Teacher Education Scheme, to be launched in September 2014, aims to attract graduates to teaching in the further education sector through bursaries and even paid training positions.²⁷⁵

Tutors may restrict themselves in terms of professional development²⁷⁶ due to their own anxieties around maths.²⁷⁶ One study noted that teachers who themselves feel uncomfortable about teaching maths, or who do not believe they have the appropriate skills, are more likely to pass maths anxiety on to their students.²⁷⁷ This suggests that training should aim to impart a high degree of confidence, as well as skills, onto potential teachers - particularly those in vocational contexts, who may not previously themselves have received a high level of maths education.

▶ **Training should provide teachers with approaches to developing student motivation.**

Studies of the field of motivation could usefully be incorporated into teacher training programmes so that teachers understand how to develop intrinsic motivation in their students.²⁷⁸ There should be a particular focus on helping teachers to assess the motivational beliefs of learners so that learning can be tailored to the needs of individual students.²⁷⁹

Certain colleges were further found to use an array of resources to support continuous professional development.²⁸⁰ One college has introduced the 'Teaching and Learning Takeaway' which provides tutors with various teaching resources to take away and share with learners.²⁸¹

In addition to the lack of teachers trained in maths and numeracy, structural issues in colleges also have an impact on the level of support available to teaching staff. One maths specialist noted that the college environment is 'very different from a school situation, where you've got a maths department or you've got learning support people who've got a team around them that they can talk to and find out what's going on, so in FE your numeracy tutors tend to be working [...] much more isolated, in separate areas'.²⁸²

▶ **Collaborative working supports learner progression and motivates teaching staff.**

Studies have shown that collaboration amongst teaching staff can support learner progression in numeracy.²⁸³ Successful methods include peer lesson observation, regularising staff meetings and ensuring collaborative planning in order that teachers can work together to plan sessions and develop resources.²⁸⁴

Collaboration between tutors was found to motivate teaching staff to find new ways of incorporating maths teaching into vocational studies.²⁸⁵ Over three quarters of respondents cited 'sharing ideas' as a preferred method of learning

from the experience and expertise of others; however, CPD trainers suggested that sharing ideas must occur within a properly structured environment in order to be effective.²⁸⁶ The Maths Fest was also found to be useful in connecting tutors to developments in maths teaching.²⁸⁷ One maths specialist suggested that a teacher forum is needed in order to expand thinking about approaches to maths teaching. They noted that *'you need to have fora for people to talk about how they can do things differently and more interestingly. A lot of teachers teach traditionally, but don't know how else to do it, which is why they need an ideas forum (with something concrete coming out of it).'* It was further suggested that there should be a support programme for teachers that *'provides models of delivery, resources, a place where teachers can exchange best practice, including professional development'*.²⁸⁸

Whole organisation approaches have also been shown to have value, in particular where they incorporate a strong numeracy strategy.²⁸⁹ The features of this holistic approach include numeracy-focused Learning Support Assistants; regular communication within and between teams; and regular impact assessments that comprise reviews of numeracy provision based on learner progress data.²⁹⁰ Kirklees College has developed an English and maths policy aimed at improving consistency, embedding clear targets for outstanding success rates, articulating staff roles and responsibilities, and outlining the entitlement of young people to develop maths skills up to level 2.²⁹¹ One study by the National Centre for Excellence in the Teaching of Mathematics reported that in institutions with a dedicated head of maths, *'the subject enjoyed a higher profile and more coherent representation'*.²⁹²

8. Key Findings

The Context

CULTURE AND CONTEXT

Looking to the UK context, findings show a negative culture of maths learning in the UK and an acceptance of failure in maths. There are, moreover, low levels of functional numeracy among young people and competence in maths is generally found by employers to be insufficient. Employer requirements range from basic maths skills to high-level conceptual abilities; many work opportunities require a grade C or above in GCSE maths. It should be noted that the success of many of the Asian countries reviewed for this study are unlikely to be replicable in the UK context due to a number of factors, including the high value placed on maths success culturally, the high level of private tuition and the excellent outcomes achieved in pre-vocational maths.

Cultural context is critical as it affects the extent to which pedagogy and practices can be transferred to other situations. Cultural and socio-economic differences within countries, for example the level of learners' family income, also significantly affect learner outcomes. The key factor influencing outcomes in vocational maths across the study countries, however, is the level to which young people gain maths skills while still in general education, suggesting the critical importance of pre-vocational maths education.

Policy Development

Within the UK context, initiative and policy fatigue is prevalent amongst practitioners; the curriculum is further felt to be outdated by maths specialists and providers. The relevance of GCSEs has been queried with reference to the disconnect between GCSE pass rates and adult numeracy skills; the relevance of GCSE qualifications to employers is also uncertain, with insufficient employer input into the curriculum. There is a distinct lack of consensus as to whether or not maths and numeracy should be separate at GCSE level; what is clear from the evidence gathered, however, is that GCSE mathematics, as currently experienced by post-16 learners, is potentially highly demotivating.

Most international policies were found to focus on the supply side rather than the demand side of maths. The success of vocational maths policies was also found to be linked to both the cultural importance given to maths as well as the success of school-level maths. Several other factors were identified as having an impact on success in maths learning in the comparator countries. Countries in which teachers have been given a high degree of autonomy over which to interpret the curriculum, for example, have tended to experience good outcomes in maths. Countries which have achieved equity in results are, further, more likely to engage successfully with all learners, including those at most risk of disengagement. The development of a framework for assessing students' mathematical competencies that can equally be understood by maths specialists, educational institutions and employers was also found to be a complicated but valuable process; using a successful existing framework can simplify the task. Finally, centres of excellence in the teaching of maths have been established in several countries. It is important to note that factors such as long study hours can mask poor policies in comparator countries.

Pedagogy and Practice in Institutional Settings

CURRICULA AND CONTENT

The content debate, in much of the literature reviewed, centres on the preference for either abstract mathematics or functional mathematics; most studies point to the value of the latter for vocational learners. Functional skills training is, in fact, generally felt to be more relevant than GCSE maths within vocational contexts; the content of GCSE mathematics is felt to be irrelevant to post-16 learners who are progressing to higher education. GCSE examination modes were also felt to be overly formal and academic, and not helpful in preparing learners for the world of work. The extent to which maths curricula should be set centrally, or indeed by government itself, is, further, a matter of debate for policy makers and practitioners. There are different levels of government input into syllabi across the countries of study.

In terms of programme content, it was noted that content should be tailored to individual needs. There are, however, restrictions on vocational practitioners in determining content. The exact form of the content also inevitably depends, to a certain extent, on the vocational subject being studied; while there may be a core of topics that are critical for life and work, occupations also have specific requirements.

DIAGNOSIS AND ASSESSMENT

Findings show the importance of, and need for, accurate and informal assessment early in the course. Assessment at the end of the course also often dictates the extent to which maths is given sufficient attention by learners and curriculum time by education providers; where maths is not part of end-of-course assessment, teaching time and attention given to it may be limited.

Skills checks and assessments vary greatly depending on the institution in question. Diagnostic assessments provide a more detailed learner profile in order that individual learning plans can be written. Some models of assessment include testing prior to the commencement of courses; pre-course testing can, however, be off-putting to learners. The accuracy of such testing has also been called into question and would also require teachers and trainers to possess high levels of diagnostic skills.

LEARNER ENGAGEMENT

Findings show that the two key demotivating factors for learners are previous negative experiences with maths learning at school and a lack of appreciation of the relevance of maths in employment and real life contexts. Learner confidence is also a key challenge in attaining mathematics skills and qualifications; the requirement to re-take GCSE maths decreases confidence further if learners are expected to learn and be assessed in the same ways as they have previously experienced. It was also found that insufficient attention is paid by vocational learning providers towards understanding the roots of learner disengagement and taking corrective measures.

Key priorities in supporting engagement include challenging maladaptive beliefs, building learners' self esteem, supporting learners to feel motivated and building a good rapport with them; intrinsic motivation should be encouraged through ongoing teacher support. Other areas supporting engagement include the learning environment, the level of parental support that learners receive and whether or not they receive appropriate careers advice and guidance. In focusing on the learning environment, promoting a focus on improvement rather than grades has had significant positive results in learner outcomes; failure should also be positively linked with the problem solving process. Learners who have previously struggled with maths were also found to engage better through the use of tailored one to one, group work and mentoring sessions. Questions should additionally be adapted to learners' cognitive levels and a mixed approach and tailoring programmes to learner needs are vitally important. Connecting maths teaching directly to learners' own lives also assists in dispelling negative preconceptions about maths. Question adaptation, relating maths concepts to learners' cultures and using teaching assistants were found to be effective mechanisms to support non-native English speakers.

PEDAGOGICAL MODELS AND APPROACHES

No single approach is appropriate for learners; approaches must be combined and tailored according to the specific needs of the learners being taught. Useful pedagogical models for the delivery of maths education in vocational settings include the Reality, Abstraction, Mathematics and Reflection model (Australia), the Discovery Math model (Canada) and the Realistic Maths Education model (Netherlands). Learning by doing, dialogic approaches and the use of innovative ways to develop thinking skills and metacognition in mathematical learning have also all been found to be effective teaching approaches for post-16 learners.

TOOLS AND RESOURCES

There were found to be a wide range of resources available to support teachers. Signposting to these resources for trainers would be useful. Technological support for learning concepts, such as video and interactive environments, is highly effective in FE and vocational teaching settings. Using e-learning is also effective, as the majority of learners are highly web-literate. The use of games and rewards are, additionally, particularly useful in re-engaging learners and building self-esteem.

EMBEDDING APPROACHES IN VOCATIONAL AND REAL-LIFE CONTEXTS

Ideas about maths education internationally have tended to move from an assumption that maths involved a set of external truths which could be communicated to learners, to a more progressive assumption that maths can be made relevant and students can be more interactive in the learning process. Contextualised learning of maths has been found to improve both the understanding of learners and the extent to which they retain the information they have learned. Learner motivation is also affected by the perception of relevance; engagement and motivation amongst learners is improved by the use of real-life contexts and examples.

Research found that vocational contexts must be authentic and relevant to the subject being studied, and should build upon the interests and experiences of the learners being taught. Learners also need to be able to move from working with familiar contexts, such as those related to their vocation, to unfamiliar contexts in order to prepare to take functional skills and GCSE assessments. Embedding should additionally be a collaborative process between vocational and maths teachers. It was noted that embedding maths in vocational contexts may be easier with functional mathematics.

Pedagogy and Practice in Employment Settings

CONTEXT AND TRAINING REQUIREMENTS

Challenges for teaching maths in the workplace include learners that may be disengaged and possess low levels of numeracy, and a lack of appropriately trained teachers. Learners need to understand the applicability and relevance of the skills they are learning to the workplace; it is also important that employers understand the maths being learnt by their work experience staff and can help to support the concepts and learning in a work context. There is greater success where maths training in the workplace is integrated into wider company strategies. The majority of employers do not have set requirements for maths skills for apprentices; the training requirements are heavily dependent on the company context. It is vital that employers work to understand the existing maths competencies of individual workers, rather than assuming a general level of competence inferred from qualifications achieved.

PRACTICE

A key finding regarding approaches to maths training in employment settings is that one delivery mode is not, and cannot be, appropriate for all learners. Maths education in vocational settings should instead be tailored to the needs of individual learners and is likely to require multiple approaches. Some employers utilise one to one support and mentoring; practical teaching methods are considered vital where training is delivered in-house. It was also suggested that being in a working environment can have a positive impact on learner confidence and motivation. Broader applications of maths beyond the workplace are also critical.

Professional Skills and Training

TRAINING OF TEACHERS AND TRAINERS

There is a significant lack of skilled teachers and trainers in vocational maths setting. Key challenges faced by teaching staff in embedding maths into vocational curricula are lack of training, lack of confidence in their own maths abilities, and insufficient teaching time. Making a GCSE qualification compulsory in a post-16 context also raises significant issues for teaching staff, who may not possess the mathematical skills to teach the curricula and prepare the learner effectively. Motivational challenges also exist for teachers who prefer sixth-form colleges, and higher paying positions, to teaching in the FE sector.

Teacher training is, therefore, a key area requiring development. It is critical that teachers and trainers possess the right level of mathematical knowledge to teach concepts effectively within vocational subjects; the provision of CPD or appropriate training for vocational teachers is vital. Training should also aim to impart a high degree of confidence, as well as skills, onto potential teachers - particularly those teaching in vocational contexts, who may not previously have received a high level of maths education themselves. Studies within the field of motivation could be usefully incorporated usefully into teacher training programmes so that teachers understand how to develop intrinsic motivation in their students. It was finally found that collaborative working motivates teaching staff and that whole organisation approaches to maths teaching are particularly important.

9. Options and Recommendations

The recommendations in this report have been developed from the collated evidence (primary and secondary) and tested with a panel of maths specialists and practitioners. A set of options, proceeding from the key findings, has been developed and are outlined as follows. It must be emphasised that the options presented are drawn from the research findings and are intended to be used to work within the current policy and practice parameters to effect positive change where required.

It is also important to recognise that a generic cure-all solution cannot be developed in order to build work-relevant maths skills in young people, due to the huge variety of requirements held by employers which can be sector-specific, level-specific and, in many cases, idiosyncratic. Similarly, borrowing from other countries' models, policies and practices can only work to the extent to which these are replicable in a UK context.

9.1 Options Development

Policy Options

Option	Possible Impacts	Risks/Challenges	Notes
Move away from the requirement for GCSE maths.	This would be likely to have the impact of increasing learner motivation, assuming that a suitable alternative is found.	A suitable alternative is needed and consensus as to the options is unlikely to be forthcoming.	This would be politically difficult to achieve and is therefore an unlikely option. An alternative may be the introduction of a level 2 core maths qualification.

Option	Possible Impacts	Risks/Challenges	Notes
Develop a committee to make recommendations on curriculum and assessment in mathematics.	This would ensure that collective expertise amongst relevant stakeholders is used to gather evidence for policymakers, with the aim of engaging learners and securing good learner outcomes.	Appointing to the committee and ensuring a balanced mix of stakeholders is vital to ensuring the right level of representation.	This committee would have significant impact and would likely proceed from the existing collaborations and working partnerships within stakeholders.
Move away from the requirement for formal assessment in post-16 vocational maths.	This is likely to support the needs of learners who do not benefit from formal assessment.	It risks practitioners dedicating less course time and learners dedicating less attention if there is no formal requirement.	Formative assessment rather than summative assessment may be a suitable compromise here; many young people are better able to demonstrate their mathematical abilities in informal and applied settings.
Develop a greater focus on functional and numeracy-focused qualifications.	As with the options above, this is likely to support learner engagement as these qualifications have greater immediate relevance and interest to disengaged learners.	A lack of focus on more abstract maths increases the risk that learners will not be able to apply their maths skills across a range of situations.	Any qualification content incorporating greater functional focus must incorporate multiple scenarios and contexts in discrete maths topics in order that learners develop applied skills in different contexts.
Give greater attention to functional and numeracy-focused content/ qualifications in pre-16 learning settings.	This may well prevent the levels of disengagement we are currently seeing in post-16 vocational maths settings, as it reduces the risk that learners lose interest in the first place.	As with the option above, there are risks attached to a lack of focus on more abstract maths.	Pre-16 maths is the key area that has the potential to influence outcomes in the post-16 vocational maths sector.
Targeting a greater proportion of funding towards more disadvantaged groups of learners.	Additional funding for disadvantaged learners, supporting a range of areas such as financing of specialist maths teachers, may have a positive influence on learner outcomes - as those from disadvantaged backgrounds are most likely to have previously disengaged from or struggled with maths.	Additional funds would be needed in a very tight funding climate. It would also require a significant amount of policy change and administrative processes to change funding streams.	Funding could be targeted in a number of ways; it might come in the form of a block grant for FE providers in the poorest fifth of areas, for example, or in a per pupil grant similar to the pupil premium. Given the associated policy changes required, however, it is unlikely that this is a viable option.
Move away from centrally determined curricula.	This would enable maths specialist organisations to play a greater role in the setting of maths curricula, making for (a) greater stability and (b) (likely) more relevant curricula.	This would be impossible to achieve under the current education system and is unlikely to gain support as an option.	This would be politically difficult to achieve and is therefore an unlikely option.

Options for Training in Institutional Settings

Approach	Possible Impacts	Risks/Challenges	Notes
Working with young people to understand their individual circumstances, e.g. family background.	Factors such as family income are correlated with the extent to which learners have negative attitudes towards maths; understanding these factors (and the distinct needs of individual learners away from these generic patterns) will help practitioners to tailor content and teaching approaches to the needs of learners.	This option assumes that practitioners will have time available to tailor content and approaches.	Ensuring that practitioners have sufficient time to understand individual learner needs is critical to the success of this approach.
Embedding the teaching of maths in vocational contexts.	Embedding in this way is likely to increase learners' engagement in maths; it may also reduce learner drop-out rates.	Tailoring content requires resources (financial and time) from practitioners. The need for tailored content limits the scalability of such approaches.	Scalability may be supported by technology as resources can be shared across institutions.
Using relevant and specific vocational scenarios to support learning, for example calculating dye quantities in a hair and beauty course, or discussing profit and loss in a business course.	Contextualising maths in ways of visible relevance to learners' chosen occupational area, can lead to better understanding and recognition of the importance of maths.	None apparent.	Developing a central repository of training tools and resources for vocational content can help practitioners apply these scenarios.
Using broader non-vocational contexts which can support learners to develop an identity through which they can view themselves as competent, for example money, employment and the law.	Use of these kinds of contexts can help learners to develop their self esteem in critical life skills areas.	As above, this requires significant resource to tailor content for non-vocational contexts,	Developing a central repository of training tools and resources for vocational content can help practitioners apply these scenarios.
Working to improve mathematical resilience among young people, for example with Dweck's mindset work (moving learners from 'I can't do maths' to 'I can't do maths yet').	Work around mindset and broader psychology has huge potential to support learners' self-belief, esteem and levels of motivation.	This option assumes that practitioners and training employers understand learner psychology and how to apply it successfully.	Developing a set of resources around improving mathematical resilience and other psychological tools will help practitioners to apply these methods.
Encouraging learners of the value of moving away from explanations towards broader learning strategies - e.g. group work and getting learners to explain ideas and solutions to peers and teachers.	This encouragement can support learners with maladaptive beliefs about maths to engage in a broader range of activities.		Learners with maladaptive beliefs about maths are also likely to believe that learning by rote is sufficient, so need additional encouragement of the value of broader learning strategies.

Approach	Possible Impacts	Risks/Challenges	Notes
Developing high expectations of learners.	This can lead to positive reinforcement, whereby students have high expectations both of themselves and their teachers; and support learners to build their self esteem.	None apparent	Relationships should be, wherever possible, supportive, respectful and equal.
Selecting an appropriate model on which to base post-16 vocational maths teaching (e.g. Realistic Maths Education or RAMR).	Use of models has been shown to have positive outcomes among learners.	Many of the models contain a dense theory base, meaning that practitioners may need additional support, resources and/or training to help them to incorporate them in lesson delivery.	Developing a toolkit to understand teaching models for post-16 vocational maths may help practitioners to apply different and more suitable models for their learners.
Using self-paced training materials, e.g. 'writeon', and other technology-supported tools and resources.	Use of existing tools and resources can save practitioners valuable preparation time and minimise duplication of effort. Use of self-paced training materials - with additional support for those with particularly low levels of literacy and numeracy - can enable learners to develop skills at their own pace and support better outcomes.	Some of these resources may require additional funding or diversion of funding.	There is a huge variety of existing resources and material available, some of which is signposted elsewhere in this report, and which would be valuable in practice.

Options for Training in Employment Settings

Option	Possible Impacts	Risks/Challenges	Notes
Integrating maths training into wider company strategies.	Maths training in the workplace is more likely to be successful when located within a wider learning strategy and linked to overall business needs.	Assumes that employers are engaged enough to make whole-business changes.	A promotional strategy focusing on the benefits to business of maths training may address this issue.
Working with individual learners to identify their actual competency level, e.g. through numeracy and aptitude tests.	The outcome of this is the ability to tailor teaching and learning to the needs of the individual.	This option assumes that employer trainers have the time and capacity to conduct competency tests. Also, formal tests may not reveal the actual abilities of learners in situations in which they have had negative experiences with formal testing.	Employers conducting in-house training should be given toolkits to apply numeracy and aptitude tests, with guidelines on engaging reluctant learners.
Demonstrating progression and/or promotion among existing employees linked to maths skills.	This can have the effect of increasing learner motivation.	None apparent	Requires employers to consider, if they do not already, maths skills requirements linked to specific job levels.

Option	Possible Impacts	Risks/Challenges	Notes
Linking training to practical business needs, e.g. measuring a radiator for installation.	Both motivation and outcomes can be improved when learners see the applied use of maths skills in the workplace.	None apparent	Developing a central repository of training tools and resources for vocational content can help trainers apply these scenarios.
Offering one to one support and mentoring to learners.	This can be of particular help to learners who may be struggling with maths elements in employment-based settings.	One to one support implies significant resource requirements but can help to enable tailored and student-specific learning.	Emphasising the business benefits of engaged workers with confidence in maths may overcome concerns about costs.

9.2 Recommendations

Policy Recommendations

► The development of a qualification leading to Core Maths level 3.

Building learner confidence and challenging maladaptive beliefs so that learners can engage with maths is vital. Ways to build confidence include mentoring, one to one support, clear progression routes and ensuring that provision is directly tailored to the learner's current level.

Although Functional Skills are viewed as stepping stone qualifications to GCSE, they were not designed for this. Currently, the level 2 Functional Skills qualification does not enable progression to GCSE maths in a structured way, nor does it lead to more applied higher level maths such as the level 3 Core Maths qualification. It is a smaller and shorter qualification, designed to be flexible and fit around the requirements of a vocational study programme.

The development of a qualification that could be completed in a modularised way as a series of small units, but which leads to a higher level qualification such as level 3 Core Maths, could provide a clearer progression route. A level 2 Core Maths qualification that draws on the applied nature and modularised route of the Functional Skills qualification, as well as the content of GCSE maths to enable progression directly onto level 3 Core Maths, may be a solution to these issues.

► The establishment of a maths expert group.

Diagnostic assessments, whether before or at the beginning of a course, are important to ensure that learners are placed on appropriate programmes and are progressing well. Many initial assessments suggest results which are significantly different from learners' existing qualifications; learners are often found to be learning at a different level than their GCSE attainment suggests. Assessments must be designed in a way which supports learners and builds their confidence levels; further research around appropriate assessment mechanisms and current innovations in the area would be worthwhile.

A maths expert group could assure the quality and robustness of such an approach. Such a body could also make recommendations for maths in the national curriculum. Changing learner mindsets towards maths is vital, and the newly established body could work towards Core Maths content that is highly engaging and that motivates learners. The expert board could also foster links with both education and training providers and employers in order to ensure that maths learning is both directly relevant for employers and that providers are afforded regular information on best practice. This expert body could also ensure that evidence-based approaches are designed to engage learners and utilise best practice, and to ensure that the needs of post-16 vocational learners are considered.

The body could also work to introduce engaging material at the pre-16 level in order that attitudinal changes are affected from a young age. Research into the development of a framework for assessing students' mathematical competencies that can equally be understood by maths specialists, educational institutions and employers could also be led by the Committee. Pre-established maths organisations would be engaged through consultations.

► Support for teachers

Additional pre-service training and CPD for teachers and trainers, both in maths and in how to contextualise maths, is likely to improve outcomes for learners. Research reviewed for this study found that specialist support for contextualisation - for example, having a maths specialist available to support contextualisation in construction training - improves outcomes. CPD requirements therefore include support for maths specialists in methods for contextualisation.

Practice Recommendations

► A central repository of teaching and learning resources and tools.

The most effective teaching approaches recognise that learners have different learning styles and competencies; there are a number of tools and resources that are highly effective. A central repository of these resources and tools could

be made available to teachers. These teaching approaches could be accumulated and disseminated to maths teachers and vocational trainers through an online portal dedicated to innovative approaches in maths teaching. This resource would list an array of effective teaching approaches including ideas about personalised teaching approaches; group work; games that could be effectively used in a number of contexts; technological support available and links to other platforms; practical exercises; and ways to effectively embed approaches in vocational and real-life contexts. The website could additionally act as a useful idea sharing forum for teachers and vocational trainers and could additionally facilitate an annual 'ideas fest' in order that teaching staff can come together and share instances of effective practice in a conference setting. The site could be overseen by the suggested maths expert group in order that approaches listed can be directly implemented in line with current curriculum content.

▶ **Collaborative working between maths teachers and vocational teachers.**

Incentives could be put in place in order that FE colleges introduce structures to ensure collaborative working between teachers, particularly between maths teachers and vocational teachers including maths in their subjects. There should be a focus on maths teaching and CPD stemming from clear leadership within colleges, as well as a dedicated maths Head or qualified maths teacher or trainer in order to enable a cross-organisational focus on maths. There should further be a focus on subject-specific CPD. It is also suggested that CPD training be held in-house where possible in order to assure attendance.

▶ **General practice recommendations:**

- **Vocational or real-life contexts should be authentic** and not feel contrived. At the same time, subjects of general interest can be useful to learners, as it enables broad-ranging discussion and engagement which might not be possible with a focus on purely vocational contexts.
- When using discussions such as these in which to embed maths, **the maths itself should not be prioritised at the expense of the discussions**, or teachers/tutors risk losing the engagement of the class - which defeats the end point.
- Situating maths in real-life situations in which learners are interested can have knock-on effects in their **recognition of potential relevance to their vocational area of study and/or their daily lives.**
- **Greater engagement can engender greater levels of confidence** among learners.

Employer Recommendations

▶ **Employer input to the maths curriculum.**

The report findings show that many employers feel that the current curriculum and qualifications available may not meet their needs and support the development of employees with the necessary skills and competencies. Encouraging employers to be more visible in their contributions to curriculum development will ensure that the skills of the workers meet sector and employer-specific requirements.

▶ **The business benefits of employee maths competence.**

Employers with apprentices could be encouraged to focus on developing maths skills in their employees, and should further be encouraged to embed maths competency within the wider business objectives. Learning maths within the workplace increases engagement and yields positive outcomes such as job progression. Employers should further be encouraged to link job levels, where appropriate, with maths competency, so that apprentices are easily able to see the benefits of pursuing maths competency and qualifications.

▶ **Signpost training toolkits for employers conducting in-house training.**

Employers conducting in-house training need signposting to the available training toolkits, such as those available from MEI. In addition, employers would benefit from the development of sector-specific training toolkits in maths skills, and resources focusing on the ways in which relevant vocational contexts may be used to enhance maths skills training.

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