Lesson plan

Area and volume

Contents

1. Rationale

Students at this level usually know how to find the area and volume of 2-dimensional and 3-dimensional shapes, and can recall and apply the relevant formulae correctly. **Valuing and building on students' prior learning** is an important part of the mastery approach (Key Principle 2). In this lesson, time is spent discussing *why* their area and volume calculations work; this establishes what students already know and supports them in developing a deeper understanding.

This lesson's focus on how scaling the dimensions of rectangles and cuboids affects the area and volume of these shapes provides an opportunity for students to see the **links between mathematical concepts** such as area and volume, proportionality, enlargement and similarity. Helping students to make **connections across the curriculum** is an important aspect of teaching for mastery (Key Principle 3).

2. GCSE curriculum

Geometry and measures

Mensuration and calculation

G19 apply the concepts of … similarity, including the relationships between lengths in similar figures

Ratio, proportion and rates of change

R2 use scale factors …

R12 compare lengths, areas and volumes using … scale factors

3. Lesson objectives

- Understand how the area and volume of shapes are affected by scaling the sides
- Understand and apply conservation of area and volume
- Use relationships between similar figures to determine areas and volumes

4. Starting points

The lesson assumes that students can apply the formulae for the area of a rectangle and the volume of a cuboid, and they have some familiarity with mathematical similarity.

5. Research questions

Pedagogic focus

How is the lesson developed and brought to a close in ways that value and build upon what students already know?

Maths focus

What evidence do you observe of students' prior learning about area and volume, and how do they work with this or modify this?

6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Introduction	20	Check students' understanding of the terms 'area', 'surface area', volume' and 'similar' and ask them to write down everything they can about the surface area of a 20 cm by 10 cm by 18 cm cuboid. Introduce the context of soft play blocks and non-slip fabric on the base of the blocks. Compare areas where one and both lengths of the base of the block are doubled. Extend to a comparison of volumes where one, two and all three dimensions of the block are doubled.	Mini whiteboards Slides 2–14
Explore 1	15	Ask students to work in pairs to complete the area and volume for the two blocks represented in the grid. Then, ask them to determine where Blocks 1, 2 and 3 belong and place them in the correct cells in the grid. Ask students to complete the numbers in the row and column headers as they work.	'Soft play blocks grid' handout, 'Soft play block sizes' cards, scissors and calculators Slides 15–16
Review/ Discuss 1	10	Check that students have correctly completed the missing areas and volumes, and students agree on where Blocks 1 to 3 belong. Discuss what students have noticed while completing the task. Identify similarities between the blocks in the grid and make comparisons to explore conservation of volume.	Slides 17–18
Explore 2	15	Ask students to consider the dimensions of blocks that will go in the blank cells of the grid.	Calculators Slide 19
Review/ Discuss 2	20	Generalise the effects of scaling dimensions on area and volume and use similarity to find areas and volumes of similar shapes.	Mini whiteboards Slides 20–27
Practice question	10	Ask students to answer a practice question and discuss their thinking.	'Practice question' handout Slide 28

7. Teacher guidance

Introduction

Aim	To establish students' existing knowledge and introduce the context
Materials	Mini whiteboards
Slides	Slides 2–14
Time	20 minutes

A key element of teaching for mastery is to **value and build on students' prior learning** (Key Principle 2). The lesson starts with identifying what students already know about area and volume, making use of arrays to help students to **develop an understanding of mathematical structure** (Key Principle 1) through mathematical representations.

What the students might do and what you might do

Slide 2 Begin the lesson by spending a few minutes checking students' understanding of the terms 'area', 'surface area', volume' and 'similar'.

Slide 3 Ask students to think about the 20 cm by 10 cm by 18 cm block in terms of *area* and write, on their mini whiteboards, everything that they know about the surface area for this block. Give students a couple of minutes to think individually and then share their thinking with a partner.

As students work, circulate and observe what they are writing on their whiteboards. If it is clear that many of them are struggling to get started, spend some time revising and reviewing areas of surfaces of 3-dimensional shapes. Students should be able to identify that the six faces of the block consist of three pairs of different-sized rectangles.

Students may calculate the *volume* of the cuboid; it is important to **value this and build on students' prior learning** (Key Principle 2) when exploring the volume of blocks later (see Slide 11).

Slide 4 Ask a couple of students to share their thinking. You may want to call on students that you identified while they were working.

If students are able to work out the areas as 200 cm^2, 360 cm^2 and 180 cm^2, check whether they can explain *why* you multiply 20 cm by 10 cm, 20 cm by 18 cm and 10 cm by 18 cm. Probing students further to gain an insight into whether they are recalling and applying a formula for the area of a rectangle, or whether they have a deeper conceptual understanding of area, is important in establishing **prior learning** (Key Principle 2).

Use the animations on the slide to help to develop an understanding of why the sides are multiplied to give the area.

Deepening understanding

- Ask students to explore total surface area, and consider how the areas discussed could be used to find the surface area of the block.
- Ask students to compare the ‘20 cm by 18 cm’ and ‘10 cm by 18 cm’ sides of the block. You might want to ask them questions such as, do they notice that the area of the ‘20 cm by 18 cm’ side is twice the area of the ‘10 cm by 18 cm’ side? Can they explain why this is so? Can they identify the dimensions of a rectangle with an area that is double the area of the base?

Slide 5 Tell students that a manufacturer of soft play equipment makes blocks of different sizes. The smallest side a block can have is 10 cm and the biggest side is 60 cm.

Slide 6 Explain to students that a piece of non-slip fabric with a logo on goes on the base of each block.

Slide 7 Introduce the smallest block that the manufacturer currently produces. The block measures 10 cm by 10 cm by 18 cm and requires a 10 cm by 10 cm piece of non-slip fabric for the base. Check that students recognise that the area of the fabric on the base is 100 cm^2.

Deepening understanding Ask students to consider what happens to the area of the base when the height of the block is increased/decreased and establish that the area of the base is not affected by the block’s height, even though the areas of the sides and the volume of the block are.

Slide 8 Explain to students that two employees, Charlie and Dee, are discussing the area of non-slip fabric needed for different sized blocks and suggesting what will happen to the area when you double one or both of the sides. Ask students to discuss in pairs which employee, Charlie or Dee, is correct. Avoid telling students who is correct, as this will be discussed on the next slide.

Slide 9 After a couple of minutes, display the three base sizes in question. Ask for a show of hands from students who think that the 20 cm by 20 cm base has double the area of the 10 cm by 10 cm base, and then from those that think the 20 cm by 10 cm base has double the area. If there is disagreement, ask a couple of pairs of students with opposing views to explain their thinking. If there is agreement, ask a couple of different pairs of students to share their reasoning. It is important to explore different ways of thinking about the areas and acknowledge different methods of explaining the relationships between the three different base sizes to **value students’ prior learning** (Key Principle 2) and to encourage a **collaborative culture** where students share their understanding (Key Principle 5).

Once it has been established that the area of the 20 cm by 10 cm base is twice the area of the 10 cm by 10 cm base, establish how many times bigger the area of the 20 cm by 20 cm base is than the 10 cm by 10 cm base.

Slide 10 Develop the discussion further to look at the features of the three different sized bases.

Ask students what is the same and what is different about the three bases. Students may comment that two of the bases are squares and one is a rectangle. They may also recognise that the 20 cm by 20 cm base is mathematically similar to the 10 cm by 10 cm base, demonstrating that they understand that similar shapes are in proportion to each other.

Ask students what they notice about the logos on the three bases. Students may comment that the logo on the 20 cm by 10 cm base has been stretched and so is no longer in proportion, whereas the logo on the 20 cm by 20 cm base is still in proportion. Using the term ‘proportion’ in their description suggests that they have made the connection between similarity and proportionality. Emphasising the **connections between mathematical concepts** is an important aspect of mastery teaching (Key Principle 3).

Emphasising similarity will be important as the lesson goes forward.

Deepening understanding Ask students to identify how the 10 cm by 10 cm base has been transformed into to the 20 cm by 20 cm base. You may need to prompt them by telling them to use words such as enlargement, stretch, reflection, and translation.

Slide 11 Use the animation on this slide to check that students can explain *why* multiplying the three dimensions of the block together gives the volume.

Spending time discussing why the calculation works is important in **establishing what students already know** (Key Principle 2). Students may explain the volume in different ways and it is important to acknowledge different ways of thinking. Providing opportunities to share enables students to help one another to succeed as a particular approach may help others to deepen their understanding, thus promoting a **collaborative culture** (Key Principle 5).

Slide 12 Display the 10 cm by 10 cm by 18 cm block and explain that the soft play equipment manufacturers are now considering a block that has one 10 cm length doubled to 20 cm while the other dimensions (10 cm and 18 cm) remain the same. Ask students to discuss in pairs what will happen to the volume of the block.

After a few minutes, ask a couple of different pairs to explain their thinking. Observe what they refer to when they explain the relationship between the two blocks and make sure to **acknowledge and value their prior learning** (Key Principle 2). Use the animation to show that two of the 10 cm by 10 cm by 18 cm blocks make up the 20 cm by 10 cm by 18 cm block, so the volume is two times bigger.

Slide 13 Remind students that the volume of the 10 cm by 10 cm by 18 cm block is 1800 cm^3. Present the two statements and ask students to work in pairs to check whether Charlie and Dee are correct. Avoid telling students who is correct, as this will be discussed on the next slide.

Slide 14 After a couple of minutes, display three blocks that fit Charlie and Dee’s criteria. (Another possible block that fits Charlie’s criteria would be a 20 cm by 10 cm by 36 cm block, whereas there is only one possible block for Dee’s criteria). Ask for a show of hands for whether Charlie is correct, whether Dee is correct, or whether both are correct. Discuss Dee’s misconception, emphasising that doubling all three lengths will make the volume 2 × 2 × 2 = 8 times bigger.

Explore 1

Aim	To explore the effects of scaling the sides of rectangles or cuboids

Materials	‘Soft play blocks grid’ handout, ‘Soft play block sizes’ cards, scissors (if the cards have not already been cut out) and calculators *The two handouts are in colour, but there is no need to print in colour, unless you wish to do so.*
Slides	Slides 15–16
Time	15 minutes

This section of the lesson includes some repeated area and volume calculations, providing an opportunity for students to develop both their **fluency** (Key Principle 4) and a deeper understanding and application of facts (multiples and factors, for example) to make **connections between mathematical concepts** (Key Principle 3). Encourage students to use a calculator where necessary, as the focus of this lesson is not on calculations.

It is intended that students should work in pairs, promoting a **collaborative culture** where students share their understanding (Key Principle 5). It is important that in their pairs, students are both able to articulate their thinking so that either student is able to explain their work.

What the students might do and what you might do

Slide 15 Distribute the ‘Soft play blocks grid’ handout and a calculator to each pair of students and explain that they are going to be working in pairs with a grid containing rows and columns. Tell students that the smallest block they have been discussing is in the top row; first column of the table (C1) because the area of the base is 100 cm^2 and the volume of the block is 1800 cm^3. Ask students to work in pairs, taking turns, to calculate the area of the base and the volume of the two other blocks in the table (C5 and C9).

Slide 16 As students are working, distribute a copy of the ‘Soft play block sizes’ cards to each pair. Students will need a pair of scissors if the cards have not already been cut out.

Check that students have understood how the table works. For example, any block in the column ‘**Area** of base is **3** times bigger’ will have a base with an area that is three times the area of the base of the 10 cm by 10 cm by 18 cm (smallest) block (C1).

Explain to students that they need to decide where to place the three blocks. They should complete the area of the base and volume of the block on each card. Encourage students to explain their reasoning to each other.

Ask students to complete the numbers in the row and column headers as they work. It might be helpful to prompt students to compare C5 and C9 initially to help them recognise that the base of C9 has an area that is three times bigger than the area of the base of C5 (and so is nine times bigger than the 100 cm^2 base).

Review/Discuss 1

Aim	To describe relationships between blocks with different dimensions, identify similarity and explore conservation of volume
Slides	Slides 17–18
Time	10 minutes

As you review the grid with students, pay particular attention to any parts of the task that you noticed pairs struggling with. Encourage different pairs to describe their thinking to identify the different approaches used. It is important to **value and make sense of students' different ways of working** (Key Principle 2).

What the students might do and what you might do

Slide 17 First, check that students have correctly worked out the missing base areas (C5 = 300 cm^2, C9 = 900 cm^2) and block volumes (C5 = 5400 cm^3, C9 = 16 200 cm^3). Ask different pairs of students to talk about how they worked out the missing information.

Use the three questions on the slide to prompt further discussion and explore the relationships between the blocks. None of the blocks are similar but the bases of C1 and C9 are similar.

Explore the conservation of volume by asking students to compare the volumes of a 10 cm by 10 cm by 54 cm block (C4) with C1, C5 and C9. For example, ask them to explain why the volume of a 10 cm by 10 cm by 54 cm block (C4) is the same as the volume of a 30 cm by 10 cm by 18 cm block (C5).

Slide 18 Check that students have placed the three blocks in the correct cells (C4 – Block 1, C8 – Block 2 and C12 – Block 3). Ask different pairs of students to talk about the block positions and headers, and explain their thinking to the rest of the class. Some may have used C8 and C9 / C9 and C12 to help them in completing the missing values in the row and column headers. Some may have noticed that Block 1 (C4) is three times taller than C1, Block 2 (C8) is three times taller than C5, and Block 3 (C12) is three times taller than C9. Discuss what effect this has on the volume. The conservation of volume can be discussed further by comparing the dimensions of C8 and C9, both of which have a volume of 16 200 cm^3.

Explore 2

Aim	To apply understanding of the effects of scaling lengths of a cuboid to determine missing lengths
Materials	Calculators
Slides	Slide 19
Time	15 minutes

In this part of the lesson, students identify blocks for the empty cells in the grid.

Slide 19 Ask students to draw in two other blocks in any of the empty cells (C2, C3, C6, C7, C10 and C11) and work out their dimensions. You may want to choose two empty cells for students to work on rather than asking them to choose. It might also be helpful to work through the dimensions of a block together, as a class, before asking students to work in pairs on this.

Deepening understanding It is likely that students will use lengths of 10 cm and 30 cm for the bases of the six remaining blocks. However, the lengths of the bases can be any factors of 100 cm^2 (C7 and C10), 300 cm^2 (C2 and C11), or 900 cm^2 (C3 and C6). Encourage students to consider other possible lengths for blocks with base areas of 100 cm^2, 300 cm^2 and 900 cm^2. C2 could, for example, have dimensions 12 cm by 25 cm by 6 cm or 15 cm by 20 cm by 6 cm. While the height will always be 6 cm, any factor pairs of 300 are possible for the other two lengths.

Asking students to consider all factors of 100, 300 and 900, and making reference to the process of identifying possible sizes of chocolate bars and trays in Lesson A can support students in making **connections between different areas of the curriculum** (Key Principle 3).

Review/Discuss 2

Aim	To make generalisations about the effects of scaling and use similarity to determine areas and volumes
Materials	Mini whiteboards
Slides	Slides 20–27
Time	20 minutes

A common misconception when scaling dimensions is that the scale factors for length, area and volume are the same. Having explored the effects on area and volume when one, two or all three lengths are doubled and trebled, students should begin to generalise. Examining the relationships between area and length scale factors, and volume and length scale factors, provides an opportunity for students to **deepen their understanding** of how the area and volume of 2-dimensional and 3-dimensional similar figures are related (Key Principle 1).

What the students might do and what you might do

Slide 20 All of the cells in the table do not need to have been completed in order to review students' work and draw together their collective thinking. Rather than revealing all the blocks and then discussing the questions posed on the slide, pose the questions at appropriate points in the discussion based on students' responses.

Discuss the dimensions for possible blocks for a couple of the spaces, focusing on the relationships between the rows and columns. For example, C2 needs to have the same volume as the 10 cm by 10 cm by 18 cm block (1800 cm^3) and a base with three times the area (300 cm^2). This means that the height needs to be divided by 3, giving 6 cm. However, if we compare C11 with C7, the height remains the same, as the volume of C11 is three times the volume of the C7.

Help students to make some generalisations about what they notice. For example, in each row, the heights of the blocks in the second column are one-third of the height of the blocks in the first column. Similarly, the heights of the blocks in the third column are one-third of the height of the blocks in the second column (and a ninth of the height of the blocks in the first column).

Check whether students recognise that C1 and C12 are the only two blocks that are similar. However, the bases of the blocks in C1 and C4 (10 cm by 10 cm) are similar to the bases of the blocks in C9 and C12 (30 cm by 30 cm) as they are an enlargement scale factor 3.

Deepening understanding Ask students to compare the areas of the sides and total surface area of the blocks (not just the base). For example, the sides of C4 (10 cm by 54 cm) and C9 (30 cm by 18 cm) have areas of 540 cm^2. The total surface area of C12 is 9 times bigger than the total surface area of C1, whereas the volume of C12 is 27 times bigger than C1.

Slide 21 Using examples from the table, remind students that:

- if one length of a rectangle is multiplied by 3, the area is multiplied by 3; the resulting shape is not similar, and
- if two lengths of a rectangle are multiplied by 3, the area is multiplied by 9; the resulting shape is similar.

Ask students what would happen to the area if the lengths were multiplied by 5, rather than 3.

Slide 22 Ask students to describe what happens to the area when both lengths are multiplied by k. You may need to remind them about the cases explored:

- when both lengths are multiplied by 2, the area is multiplied by 4 (slide 9),
- when both lengths are multiplied by 3, the area is multiplied by 9 (slide 18),
- when both lengths are multiplied by 5, the area is multiplied by 25 (slide 21).

Check that students recognise that 4, 9 and 25 are square numbers.

Encouraging students to apply their existing knowledge of different types of numbers helps to **make the links across mathematics explicit** to them (Key Principle 3).

Slide 23 Remind students about the relationship between the volumes of the blocks when one length, two lengths and all three lengths are doubled.

Slide 24 Establish the effect on the volume of a cuboid when one, two and all three lengths are multiplied by k.

Slides 25–27 These slides can be used to consolidate student learning and emphasise how the relationships between similar figures enable us to find areas and volumes without the need to calculate. It is important to emphasise that we can use similarity to determine area/volume when we don't have enough information to calculate the area/volume using dimensions, provided we know the area/volume of one of the similar shapes. Ask students to show their work on mini whiteboards.

Slide 25 Ask students to work out the area of the regular pentagon with side 10 cm, using what they know about the area of the similar regular pentagon with side 5 cm. (Area = $43 \times 2^2 = 43 \times 4 = 172$ cm^2)

Slide 26 Ask students to work out the volume of the second triangular prism, using what they know about the first one. (Volume = $120 \times 3^3 = 120 \times 27 = 3240$ cm^3).

Slide 27 Check that students recognise that the two leaves are similar and ask them to identify the scale factor of enlargement. Once it is established that the first leaf has been enlarged by a scale factor of 2, ask students to determine the area of the second leaf. (Area = $9 \times 2^2 = 9 \times 4 = 36$ cm^2).

Now use the volume example to highlight that you can look at the relationship in both directions, i.e. where the shape has been made smaller rather than bigger. Discuss different ways of working out the volume. For example, do students consider a scale factor of one-third and multiply by $\frac{1}{27}$, or do they prefer to think of it as dividing by 27 to get a volume of 4 cm^3?

Practice question

Aim	Students apply their knowledge to a practice question
Materials	'Practice question' handout. It is not necessary to print this out: the question can be displayed on the board.
Slides	Slide 28
Time	10 minutes

In this part of the lesson, students apply what they have learned in the lesson to a practice question. Give students a couple of minutes to work on the question individually and then discuss their approaches.

What the students might do and what you might do

Slide 28 Check that students have correctly identified the length scale factor of 2. They should be able to use the volume scale factor ($2^3 = 8$) to determine the volume of solid B.

When finding the surface area of solid A, they should recognise that the surface area is the sum of the areas of the sides and so the area scale factor ($2^2 = 4$) applies. They should realise that A is smaller than B and so they need to divide by 4 or multiply by $\frac{1}{4}$.