Lesson plan

Basic algebra

Contents

1. Rationale

Students often view a letter in algebra as representing a specific unknown that needs to be found and find it difficult to answer questions in which the unknown is a variable. This lesson starts by introducing the context of buying a carpet with width 4m and variable length n to establish that n represents a variable. This provides students with a 'concrete' model that supports the **'concrete, pictorial, abstract' (CPA) approach** to teaching for mastery.

Later in this lesson, area models are used to highlight the relationship between algebraic expressions in factorised and expanded form, and to support students in understanding multiplicative algebraic structure. Developing an understanding of **mathematical structure** (Key Principle 1) through mathematical representations is a key part of the teaching for mastery approach.

This lesson's focus on what it means to fully factorise an expression helps to establish links with common factors and highest common factors. This encourages students to see the **links between mathematical concepts** (Key Principle 3).

2. GCSE curriculum

Notation, vocabulary and manipulation

A4 simplify and manipulate algebraic expressions (including those involving surds) by:

- collecting like terms
- multiplying a single term over a bracket
- taking out common factors
- expanding products of two binomials

3. Lesson objectives

- Use correct algebraic notation
- Expand brackets
- Factorise algebraic expressions
- Understand multiplicative algebraic structure using an area representation

4. Starting points

The lesson assumes that students have some knowledge of basic algebra: they should understand that a letter in algebra represents an unknown, and they should know how to simplify basic algebraic expressions.

5. Research questions

Pedagogic focus

How does the teacher promote understanding of mathematical structure?

Maths focus

In what ways do students use representations to consider mathematical structure to develop their mathematical understanding?

6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Introduction	15	Introduce the context of buying carpet. Discuss how to describe the area of the carpet.	Mini whiteboards Slides 2–6
Explore	35	Ask the students to work in pairs to complete area models for a set of sequenced expressions. Then ask them to use the completed area models to help identify the factorised and expanded forms of each expression.	'Algebraic expressions' grid 'Expressions' cards Scissors (if the cards have not already been cut out) Sticky notes (*optional*) Slides 7–9
Discuss	10	Discuss the completed task. Check that students understand the answers, particularly focusing on those expressions that you noticed they struggled with.	Mini whiteboards Slides 10–11
Review	20	Discuss the 'Algebraic expressions' grid as a whole. Address common misconceptions and draw out what it means to factorise an expression fully.	Mini whiteboards Slides 12–17
Practice question	10	Ask students to answer an exam question and then discuss their methods.	'Practice question' handout Slide 18

7. Teacher guidance

Introduction

Aim	To introduce students to context and area representations
Materials	Mini whiteboards
Slides	Slides 2–6
Time	15 minutes

A key element of teaching for mastery is the **'concrete, pictorial, abstract' (CPA) approach**, in which students develop their understanding of abstract mathematical concepts and structures, starting with concrete objects, moving on to pictorial representations and then linking to abstract mathematical symbols. This section of the lesson starts with the context of buying carpet, which students should be able to visualise. Within this context, n (used to describe the length of carpet a customer requests) represents a variable.

Another Key Principle of the mastery approach is teaching that allows students to develop an understanding of **mathematical structure** (Key Principle 1). Using an area diagram as a pictorial representation of the carpet exposes the relationship between an expression given in both expanded and factorised form.

What the students might do and what you might do

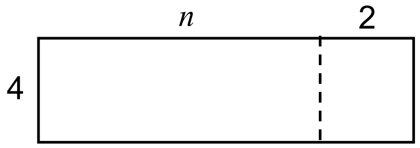
Slide 2 Help students to understand the context. Tell students that Sam is buying new carpet and has chosen a carpet that comes in a 4-metre width. The owner of the shop uses the length of carpet a customer needs to find the area of carpet being bought, so he can work out the cost. Explain that n is used to represent a variable length as the length of carpet being bought varies from customer to customer.

Slide 3 It is important to spend enough time on this slide to emphasise that n is a variable. When working with an unknown quantity, n, students may assume that the value of n needs to be found. After showing the animation, hold a short discussion about what n could be. Emphasise that we don't know what n is and make sure that students recognise that n can represent different values.

Students might describe the area of the carpet as '4 times n' and write this as $4 \times n$. It is important to emphasise correct algebraic notation and for students to recognise that the multiplication symbol is not needed when writing algebraic expressions.

Establish that length is measured in metres (m) and area is measured in square metres (m^2).

Slide 5 This slide introduces an area diagram.



Tell students that the owner of the carpet shop recommends that Sam buys an extra 2 metres of carpet (to allow for error when fitting the carpet). Check that students are happy that the diagram shows that the area calculation needed is $4(n + 2)$.

Tell students that Sam says the calculation should be $4n + 8$. Ask students if they can explain Sam's thinking.

Establish that the owner and Sam are both describing the same thing but in different ways. The owner is describing a calculation that involves multiplying two lengths, whereas Sam is describing a calculation that involves adding two areas. Emphasise where the two expressions are featured in the diagram.

Check that students understand the vocabulary of factorising. Establish that:

- 4 and $(n + 2)$ are factors
- $4(n + 2)$ is the expression in factorised form
- $4n + 8$ is the expression in expanded form.

The discussion of the area diagram will provide insights into their current understanding and allow you to **build on their existing knowledge**, which is an important part of the mastery approach (Key Principle 2).

Slide 6 Establish that the owner's expression is the expression $4n + 8$ in factorised form and Sam's expression is the expression $4(n + 2)$ in expanded form.

Explore

Aim	To use area model representations to develop understanding of expanding and factorising expressions
Materials	'Algebraic expressions' grid and 'Expressions' cards, scissors (if the cards have not already been cut out), (*optional*) sticky notes
Slides	Slides 7–9
Time	35 minutes

This section of the lesson uses area models and procedural variation to **expose the underlying structure of the mathematics** (Key Principle 1). The ten expressions in the 'Algebraic expressions' grid have been carefully sequenced by varying one thing at a time. Using an area model directs students' attention to the fact that factors are the sides and expanded form is the area, and highlights the relationship between algebraic expressions in factorised and expanded form. This helps students to avoid common errors such as multiplying the first term by the term outside the bracket but omitting to multiply the second term.

Directing students to take turns when completing the algebraic expressions grid encourages a **collaborative culture** where students work together and share their understanding and provides opportunities for students to explain their thinking to one another (Key Principle 5).

What the students might do and what you might do

Slide 7 Ask students to work in pairs and give each pair a copy of the 'Algebraic expressions' grid. Ask them to fill in the missing information on each diagram.

Observe students as they complete this task but do not intervene unless necessary. Notice which areas they find more difficult, to inform the discussion in the next section of the lesson. If a pair of students is struggling to get started, check that they understand that the sides need to be multiplied to give the area, and link this task back to the carpet model.

Depending on your class, you may decide to get them started, then discuss the answers (see Slide 8) and then ask them to continue, before going through the rest of the answers. You may also suggest that they leave out the last two diagrams.

Slide 8 Use this slide, if you need it, to support a discussion about what should go in the empty boxes on the diagrams.

Slide 9 Give each pair a copy of the 'Expressions' cards. (Students will need scissors to cut the expressions cards up if they haven't already been cut into cards prior to the lesson. Alternatively, they can copy the cards onto sticky notes). Point out that the column on the left of the diagrams has the heading 'Factorised' and the one to the right has the heading 'Expanded'. If needed, remind them that, in the example of Sam buying carpet, $4(n + 2)$ is the factorised form of the expression and $4n + 8$ is the expanded form.

Ask students to use the completed area models to help identify the factorised and expanded forms of each expression, and to use this information to place the cards in the appropriate cells. Tell them that once the cards are placed, they should write the missing expressions in any empty cells.

Deepening understanding Ask students to examine their completed table to see what they notice. Do they recognise that the expanded expressions in rows D and G are the same and yet the factorised expressions in these rows are different? Can they explain why this is?

Discuss

Aim	To make sure that students have the correct answers, and they understand why the answers are correct.
Materials	Mini whiteboards
Slides	Slides 10–11
Time	10 minutes

It is important that students recognise the relationship between the factorised and expanded forms of an expression and how factorising an algebraic expression is the opposite process of expanding brackets. Discuss the completed task and ensure that all students understand the answers, particularly focusing on those that you noticed they struggled with such as the expressions in rows I and J, $(n + 2)(n + 3)$ and $(n + 3)^2$.

What the students might do and what you might do

Slide 10 Go through the 'Algebraic expressions' grid line by line and draw students' attention to each area model and how the factorised and expanded expressions relate to the model.

Ask students which cards they were able to match first, after completing the missing information on the diagrams. They should have identified E2 as the factorised expression for the diagram in row A and E6 as the expanded expression for the diagram in row D.

Slide 11 Ask students how they completed rows I and J. How did they work with the area model? Did they view the diagram as two rows stacked together? Matching E7 to the diagram in row J relies on understanding that $(n + 3)^2 = (n + 3)(n + 3)$.

Review

Aim	To review students' understanding expanding brackets, and to establish links between factorising fully and highest common factors
Materials	Mini whiteboards
Slides	Slides 12–15
Time	20 minutes

This section of the lesson considers the 'Algebraic expressions' grid as a whole to review what was difficult and what was interesting.

A common misconception when squaring a binomial is to square each term and add. Slides 12–15 address this misconception the use of an area model representation supports students in developing their **understanding of 'why'** (Key Principle 1).

By looking at the grid as a whole, students may notice that that the expanded expressions in rows D and G are the same but the factorised expressions are different. This can be used to lead on to a discussion of what it means to fully factorise an expression. Identifying common and highest common factors within the context of algebraic expressions supports students in making **links between different areas of the curriculum** (Key Principle 3).

What the students might do and what you might do

When reviewing the table, draw students' attention to the way that the expressions in factorised form vary from one row to the next. What effect does this change have on the expanded expression? What effect does this change have on the area of the rectangle?

Ask students which rows they found the most difficult. This is likely to be rows I and J, $(n + 2)(n + 3)$ and $(n + 3)^2$, and will lead into the discussion on expanding $(n + 4)^2$ on the next slides.

Slides 12–13 Having identified that $(n + 3)^2 = (n + 3)(n + 3)$ from the 'Algebraic expressions' grid, students should be able to explain why Tony's solution is incorrect. Ask students to draw an area model on their whiteboards to work out the correct expanded expression. Check that students recognise how to get the term $8n$ (i.e. $4n + 4n$) and understand how the complete diagram relates to the final (expanded) expression.

Before moving on to slide 14, ask students to look at all the rows on the grid. What is the same? What is different? Allow time for this discussion. If this hasn't come up as part of the discussion, then ask students to compare the expressions in rows D and G.

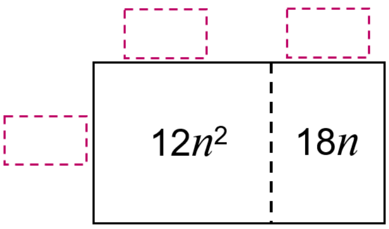
Slides 14–15 When factorising an expression, students often leave common factors inside the bracket. The factorised expression in row D, $n(2n + 2)$, is an example of this and can be contrasted to the factorised expression in row G, $2n(n + 1)$, which has been factorised fully. Did students notice that the expanded expressions in rows D and G are the same but the factorised expressions are different? Establish that an expression can be factorised in more than one way if the terms have more than one common factor.

Ask students if $2n^2 + 2n$ can be factorised another way: ($2(n^2 + n)$ where a factor of 2 is taken out).

Explain that our aim when factorising is to factorise fully. This means that there are no common factors for the terms inside the bracket. Establish that factorising fully gives $2n(n + 1)$.

Slide 16 Ask students to imagine that they are the teacher. Give them a few minutes to discuss in pairs how they could help Cameron to understand why the expression hasn't been factorised fully. Choose a few of pairs of students to share their explanations with the rest of the class.

Slide 17 Ask students to work in pairs to complete the diagram on their mini whiteboards.



After a few minutes ask a pair of students to describe their labels for the diagram. Do other students agree? Have they factorised fully? How do they know?

What are all the possible ways of factorising the expression? Can students find all six?

$2(6n^2 + 9n)$, $2n(6n + 9)$, $3(4n^2 + 6n)$, $3n(4n + 6)$, $6(2n^2 + 3n)$, $\mathbf{6n(2n + 3)}$.

Work with the class to establish that the expression factorised fully is $\mathbf{6n(2n + 3)}$.

Deepening understanding

- Can students draw a diagram to represent the expression $n^2 + 4^2$?
  Is there more than one way?
  How does the diagram for $n^2 + 4^2$ relate to the diagram for $(n + 4)^2$?
- Can students make links with prime factorisation to determine the highest common factor?
  If we express $12n^2$ as $2 \times 2 \times 3 \times n \times n$ and $18n$ as $2 \times 3 \times 3 \times n$, how does this help us to factorise the expression $12n^2 + 18n$ fully?

Practice question

Aim	Students apply their knowledge to an exam question
Materials	'Practice question' handout. It is not necessary to print this out: the question can be displayed on the board.
Slides	Slide 18
Time	10 minutes

Students apply what they have learned earlier in the lesson to an exam question.

What the students might do and what you might do

In all previous parts of this lesson, n has been used to represent a variable. Ensure that students understand that a variable can be represented by any letter.

Notice whether students draw a diagram to support their thinking. Can they explain their approach when factorising the expression in part (b)? How are they convinced that they have factorised the expression fully?

