Lesson plan

Percentage change and best buys

Contents

1. Rationale

Students often view percentage change as an additive process, calculating the percentage change amount and adding or subtracting to give the required percentage increase/decrease. This lesson focuses on the opportunity that percentage change problems provide to explore the **underlying multiplicative relationship** (Key Principle 1) between the original and new values. Considering additive approaches **that students are already familiar with** (Key Principle 2), alongside strategies that involve multiplicative reasoning supports students in **developing both their fluency and understanding** (Key Principle 4) as they learn to recognise when and how to apply additive and multiplicative approaches.

2. GCSE curriculum

Ratio, proportion and rates of change

R9 interpret percentage changes … multiplicatively; work with percentages greater than 100%; solve problems involving percentage change.

3. Lesson objectives

- Become fluent at working with percentage change
- Determine the best deal following a percentage change
- Understand different approaches to solving multi-step percentage problems
- Use representations to provide insight when solving problems

4. Starting points

The lesson assumes that students can use both non-calculator and calculator methods for finding percentages of an amount.

5. Research questions

Pedagogic focus

In what ways do teachers use diagnostic and formative assessment to support students in becoming fluent?

Maths focus

In what ways do students develop their understanding and fluency?

6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Explore/ Discuss 1	20	Introduce the packing tape context and ask students to find the length of tape on a roll that is 50% longer than 120 metres. Explore different ways of calculating percentage increase and how these approaches can be represented.	Mini whiteboards 'Approaches' handout (optional) Slides 2–11
Explore 2	10	Tell students that a 180-metre roll of brown tape is currently priced at £2.70. Ask students to find the new length after an increase of 20% and the new price after a decrease of 20%.	Calculators Slide 12
Discuss 2	10	Discuss students' work, valuing both additive and multiplicative approaches to increasing and decreasing by 20%.	Slides 13–14
Explore 3	25	Ask students to find the price per metre for a 120-metre roll of tape priced at £2.40. Use the 'Clear tape offers' handout to explore different offers involving either increasing the length of tape on a roll, or decreasing the price. Ask students to determine how the offers compare in terms of price per metre.	'Clear tape offers' handout Large paper Scissors (optional) Slides 15–17
Discuss 3	10	Discuss students' approaches to calculating percentage increase and decrease and establish what the price per metre means for both the company and the customer.	Slides 18–24
Practice questions	5	Ask students to work individually to answer two practice exam questions.	'Practice questions' handout Calculators (optional) Slide 25
Discuss 4	10	Discuss students' strategies for solving the two problems and how the multiplicative relationships relate to additive approaches.	Slides 26-28

7. Teacher guidance

Explore/Discuss 1

Aim	To explore different ways of finding a percentage increase, including both additive and multiplicative approaches
Materials	Mini whiteboards, ‘Approaches’ handout (optional)
Slides	Slides 2–11
Time	20 minutes

This section of the lesson introduces the context of three employees working for a packing tape company. Two rolls of tape produced by the company are described in terms of the length of tape on one roll in relation to the other. Teachers use diagnostic questioning to **establish students’ existing knowledge**, which is an important part of the mastery approach (Key Principle 2), and students are given the opportunity to calculate a percentage increase for themselves, before considering three approaches that involve additive and multiplicative thinking. The use of representations provides students with a means to develop their **understanding of mathematical structure** (Key Principle 1) and **make connections** between mathematical topics (Key Principle 3).

What the students might do and what you might do

Slide 2 Tell students that a company produces 120-metre rolls of clear tape and rolls of brown tape that contain 50% more tape than the rolls of clear tape. Ask students how long the rolls of brown tape are. Make a note of the different approaches adopted by students.

Slide 3 After a couple of minutes tell students that Nel, Reuben and Saskia work for the company that manufactures tape. They have each used a different approach to determine the length of brown tape. Ask students to discuss in pairs what each employee has done. Encourage them to draw diagrams to explain how each approach works. The diagrams used by Nel, Reuben and Saskia are discussed later, so do not discuss them at this stage.

Slides 4–6 Use these slides to support a discussion of the three approaches. As you discuss the different approaches, establish that Saskia has used an additive approach (100% + 50%) to find 150%, whereas Nel and Reuben are thinking multiplicatively (10% × 15 = 150% and 100% × 1.5 = 150%). Draw on your earlier observations to point out which of your students has used each approach.

Slide 4 Building up from 10% is a common method that students use when using an additive strategy and Nel may have had this in mind when finding 10% initially. However, Nel has taken a multiplicative approach, scaling up to 150% by multiplying by 15.

Slide 5 Reuben used a one-step multiplier method for finding the length of brown tape. He recognised the multiplicative relationship between 150% and 100% and identified 150% ÷ 100% = 1.5 as a multiplier representing the percentage increase.

Slide 6 Saskia has used an additive approach, dividing by 2 to find 50% (60 metres) and then adding this to the original 120 metre length. It is important that students develop non-calculator strategies such as this when working with percentage change. However, using an additive approach can often be inefficient and result in mistakes (such as omitting to add to (or subtract from) the original amount). Whilst discussing Saskia's approach, check whether students can explain why Saskia is interested in working out 150%.

Slide 7 Establish that increasing by 50% means that we add 50% of the original value to itself. The original value is 100%, so we add 50% to 100% to give 150%. Check the students' understanding by asking them what percent we would be looking for after a 20% increase or decrease.

Slide 8–11 Tell the students that Nel, Reuben and Saskia each drew a diagram to explain their thinking. The three approaches, and three diagrams are provided on the optional 'Approaches' handout. Either use the handout or go straight to slides 9 to 11: either way allow the students some time to think in pairs about which diagram might have been drawn by each of Nel, Reuben and Saskia. Where appropriate, refer to any diagrams your students drew at the start.

Slide 9 Reuben has used the relationship between 100% and 150% to obtain a multiplier for the percentage change (150% ÷ 100% = 1.5). He has then applied the multiplier to the length of the clear tape, multiplying 120 metres by 1.5 to get 180 metres.

Slide 10 Saskia has used two bars to represent the different tapes. The bar representing the clear tape has been divided into 2 equal parts (120 metres ÷ 2 = 60 metres (50%)) and the bar representing the brown tape is 60 metres (50%) longer than the clear tape bar.

Slide 11 Nel has used two bars to represent the different tapes. Both bars have been partitioned to show 10% of the original length (12 metres). The bar representing the brown tape extends beyond the length of the clear tape bar, with the total length labelled as 180 metres = 12 metres × 15.

Explore 2

Aim	To calculate a percentage increase and decrease
Materials	Calculators
Slides	Slide 12
Time	10 minutes

In this part of the lesson, students are presented with two offers to explore: one involving an increase in the length of tape on a roll and the other involving a reduction in price. Having **established what students already know** in the previous part of the lesson (Key Principle 2), students have an opportunity to **apply their**

newly developed understanding of multiplicative relationships, alongside their previous understanding, to determine the percentage changes (Key Principle 4).

What the students might do and what you might do

Slide 12 Tell students that a **180-metre** roll of brown tape is currently priced at **£2.70**. The packing tape company are investigating two possible special offers: increasing the length of tape by 20% or decreasing the price of the tape by 20%.

Ask students to spend a few minutes working individually to find the new length of tape in Offer 1 and the new price in Offer 2. Encourage them to draw a diagram to help to explain their thinking. Once students have had some time to work alone, ask them to discuss their thinking with their partner. Avoid intervening as they work: it is important that they have enough time to think before you explain. Make a note of the different approaches adopted by students, as you may want to call on students who used different approaches in the next section of the lesson.

Deepening understanding If students have calculated the new length and new price (216-metre roll for £2.70 and 180-metre roll for £2.16), ask them which of the two offers will be best for the manufacturer.

Discuss 2

Aim	To use diagrams to represent mathematical structure, and use additive and multiplicative approaches to find a percentage change
Slides	Slides 13–14
Time	10 minutes

This section of the lesson discusses the percentage problems that students worked on in the previous section. Representations are used to support students in **understanding the mathematical structure** of percentage problems (Key Principle 1), and to help students to recognise different possible approaches.

What the students might do and what you might do

Slides 13–14 Hold a class discussion, asking a couple of different students to share their thinking for the two percentage problems. Many students are likely to use an additive approach for both problems. Use the representations of the additive approaches on these two slides to support a discussion of students' additive approaches. Use the representations of the multiplicative approaches on these two slides to support students' understanding that multiplying by 1.2 gives an increase of 20% and multiplying by 0.8 gives a decrease of 20%.

For Offer 1, it is likely that students will have found 20% of 180 m (36 m) and added it on to get 216 m. In the second problem, it is likely that students will have found 20% of £2.70 (54 pence) and subtracted it to get £2.16. Check whether anyone used a multiplicative approach, using a calculator to multiply 180 by 1.2. Whilst an additive approach is easier to complete numerically without access to a calculator, the double number line representation of a multiplicative approach can be used to support students' understanding that multiplying by 1.2 gives an increase of 20%, regardless of whether students used a multiplicative approach or not.

For Offer 2, it is likely that students will have found 20% of £2.70 (54 pence) and subtracted it to get £2.16. Ask students to explain how they found 20% (did they find 10% and double it, or did they divide by 5?). Check whether any students used a multiplicative approach.

Check that all approaches used by students have been explored. It is important to **value and make sense of students' different ways of working** to increase students' self-belief (Key Principle 5).

Discuss when you might use an additive approach and when you might use a multiplicative approach. Whilst an additive approach may be less efficient, it is easier to complete numerically without a calculator.

Explore 3

Aim	To examine price per metre as a means of comparing offers involving percentage change
Materials	'Clear tape offers' handout, large paper & scissors *or* 'Clear tape offers landscape' handout photocopied onto A3 paper
Slides	Slides 15–17
Time	25 minutes

In this part of the lesson, students explore offers suggested by two of the employees. Providing the offers for the students, rather than asking them to determine them themselves, helps to **build student confidence** by giving them a starting point (Key Principle 5).

Slide 15 Tell students that a **120-metre** roll of clear tape is currently priced at **£2.40**. Ask them to calculate the price per metre.

After a minute or so, ask students to explain their thinking when finding the price (2 pence per metre). Use formative assessment approaches to check that students know which number to divide by, and why; if any students are confused, you may want to spend some time with them to establish what they do, and do not, understand.

Discuss how the price per metre affects the company and the customer. (The greater the price per metre, the more money the company get per metre, but the more expensive it is for the customer to buy, so it is better for the company. The smaller the price per metre, the less money the company get per metre and the cheaper it is for the customer to buy, so it is better for the customer).

Slide 16 The packing tape company have asked Nel and Reuben to suggest possible special offers for the clear tape. The company wants to find offers where increasing the length of tape and decreasing the price of the roll results in the *price per metre of tape being the same for the two offers*.

Check that students understand the effects of increasing the length of tape/decreasing the price. You could ask: *Will increasing the length of tape you get* *for £2.40 increase or decrease the price per metre? Will decreasing the price for 120 metres of tape increase or decrease the price per metre?*

Slide 17 Give each pair of students a large sheet of paper and scissors and a copy of the 'Clear tape offers' handout (or the 'Clear tape offers landscape' handout photocopied onto A3 paper).

Explain that Nel and Reuben have come up with some suggestions of offers for the students to explore. Before setting students off on the task, you may like to spend a couple of minutes discussing with students some of the things that might be helpful to consider. These might include things like:

- the new length of tape
- the new discounted price
- similarities and differences between Nel's and Reuben's suggestions
- a comparison of the price per metre of tape in pence
- which offer is best for the company/customer

Ask students to work in pairs to explore Nel's and Reuben's suggestions. They should write information about the offers and any calculations next to the relevant suggestion. Emphasise that there is no one correct response; there are many different, valid things they could write down. Students may prefer to use a pencil to record their thinking, so they can make any necessary amendments as they work. They could also use post-it notes.

For students to complete the activity fully, they need to work out the multiplier or the amount to add on (or subtract), calculate the new value and work out the new price per metre. Some students may need scaffolding, but do not intervene unless they are really stuck. If you do need to help them, think about how you can do this without removing the opportunity for them to work things out in their pair for themselves.

Some students may not get as far as working out the new price per metre. You will need to decide for yourself when it is appropriate to move on, even if all students have not quite completed the task.

As you circulate around the class, observe how students are working and listen to their explanations to each other. You can use your observations to inform the discussion in the next part of the lesson; this is an important aspect of formative assessment.

Discuss 3

Aim	To review different approaches to finding a percentage change
Slides	Slides 18-24
Time	10 minutes

The aim of this lesson is to **develop students' fluency and understanding** when solving percentage change problems (Key Principle 4). The double number lines on slides 18, 19, 21 and 22 help to **expose the multiplicative structure** of percentage change (Key Principle 1) and can be used to support students' understanding and appropriate use of non-calculator additive and multiplicative approaches.

What the students might do and what you might do

Slides 18-22 Use these slides to discuss Nel's and Reuben's offer suggestions in turn. Whilst students should have had access to a calculator when completing the task, they may have used non-calculator additive strategies. Encourage students to think multiplicatively as well as additively, using the representations to establish that finding 60% and adding it on is the same as multiplying by 1.6 and that finding 40%/35% and subtracting is the same as multiplying by 0.6/0.65.

Slide 18 When discussing each offer, encourage students to identify the percentage they are interested in after the percentage change. For example, for Offer A (and Offer C), an increase of 60% means that we are interested in 160%. Discuss students' approaches to finding 160%, emphasising how additive and multiplicative approaches relate to each other.

Slide 19 Students who used an additive approach may have calculated 40% of £2.40 (96p) and used this as the discounted price, rather than £2.40 – 96p = £1.44. It is important to establish the need to subtract from 100% when calculating a percentage decrease. The double number line representation can be used to support a discussion of the relationship between a percentage decrease of 40% and calculating 60%.

Slide 20 Ask students to explain how they checked whether the price per metre of tape is the same for Offers A and B. Students sometimes get confused about which number to use as the divisor in these calculations; check that they understand that price per metre means that the price for one metre needs to be found. Students may have worked in pounds rather than pence. Check that students are able to interpret a price of £0.0125/£0.012 per metre as 1.25 pence/1.2 pence per metre.

Slide 21 Offer C is the same as Offer A.

Slide 22 Using an additive approach for a percentage decrease of 35% is a more onerous process than finding 40% and subtracting (as in Offer B). If students have used an additive approach, discuss their methods for finding 35%. Recognising that we are interested in 65% may have prompted students to use non-calculator strategies to find 65%, rather than finding 35% and subtracting. Identifying the multiplier as 0.65 emphasises the need to subtract from 100% when carrying out a percentage decrease.

Slide 23 The price per metre for Offer C is already known (as it is the same as Offer A). Establish the price per metre for Offer D is 1.3 pence.

Slide 24 Remind students that the packing tape company wanted to find offers where increasing the length of tape and decreasing the price of the roll results in the *price per metre of tape being the same for the two offers*. Establish that whilst the prices per metre are similar for Offers A and B (1.25p and 1.2p) and Offers C and D (1.25p and 1.3p), they are not the same. Students may notice that there is a difference of 0.5 pence between the prices per metre for both Offers A and B and Offers C and D.

Deepening understanding Nel and Reuben were not able to find offers where the price per metre is the same regardless of whether the length of tape on a roll is increased or the price is reduced. Is it possible to find offers where this is the case? (e.g. an increase in tape length of 60% and a decrease in price of 37.5%). Why does this work? (Because 1.6 (8/5) × 0.625 (5/8) = 1) Can students identify other percentage increases/decreases that result in the product of the multipliers being equal to 1?

Practice questions

Aim	Students apply their knowledge to exam questions
Materials	‘Practice questions’ handout, Calculators (optional)
Slides	Slide 25
Time	5 minutes

In this part of the lesson students apply what they have learned in the lesson to two practice questions.

What the students might do and what you might do

Slide 25 Distribute a copy of the ‘Practice questions’ handout to students and give them a couple of minutes to come up with a solution to the two problems.

Both questions are taken from non-calculator papers. However, consider allowing students access to a calculator. It is important that students can apply both non-calculator additive and calculator multiplicative approaches. Students need to **develop fluency** (Key Principle 4) when using a calculator and in making the decision of when to use mental, written or calculator methods when they have the choice.

Discuss 4

Aim	To explore different approaches to solving problems
Slides	Slides 26–28
Time	10 minutes

A key element of teaching for mastery is to help students to develop an understanding of **mathematical structure** through the use of representations (Key Principle 1). In this final part of the lesson it is important to examine students’ use of diagrams when solving the two problems. The double number lines can be used to emphasise the relationship between additive and multiplicative approaches. **Valuing and making sense of students’ different ways of working** helps to increase students’ self-belief (Key Principle 5) and is an important aspect of mastery teaching.

What the students might do and what you might do

Slide 26 Discuss students’ thinking when finding the cost of a box of cereal at Food Mart. Encourage different students to describe their approaches and share any diagrams/representations to explain their thinking. Use the double number line representation to emphasise the relationship between reducing by 20% and multiplying by 0.8.

Slide 27 Discuss methods for finding the cost of a box of cereal at Jan's Store in a similar way. The double number line representation can be used to emphasise the relationship between increasing by 30% and multiplying by 1.3.

Discuss how students determined which offer provides the best value for money (Jan's Store). Did they find the price per gram (1p for Food Mart and 0.96p for Jan's Store) or did they identify that with Food Mart you get 400g for £4 whereas with Jan's Store you get 520g for £5 (500g would be comparable with Food Mart).

Slide 28 Students should be able to find the price of 25 plants at Kirsty's plants. Discuss what 'plus VAT at 20%' means and establish that this means finding 120%. Once students have done this, comparing the total price is straightforward.